

## Contextual Activities to Drive Basic Mathematics

The following activity, “Estimating Animal Populations,” is an example from a collection of activities written for the Basic Mathematics course by Nancy Crisler of Washington University-St. Louis and Gary Simundza of Wentworth Institute of Technology in Boston, and was included in a workshop at the 2009 NADE conference in Greensboro.

The activity is used to introduce the concept of proportion, as well as illustrating the way fundamental mathematics finds application in the real world. Such activities can comprise the framework for a new approach to the Basic Mathematics course that engages students with authentic and interesting problems from the workplace.

The authors of these activities will present an intensive 5-day workshop in June under the auspices of the Mathematical Association of America’s Professional Enhancement Program (PREP) in Washington, DC, entitled “Arithmetic in College: Revitalizing the Basic Mathematics Course.” The workshop will explore the activity-based approach in depth, and will include discussions regarding strategies for implementing such an approach. For more information, see [www.teachmathapplications.com/maaprep.htm](http://www.teachmathapplications.com/maaprep.htm).

For more information about the activities themselves, contact Gary Simundza, [simundzag@wit.edu](mailto:simundzag@wit.edu), 617-989-4354.

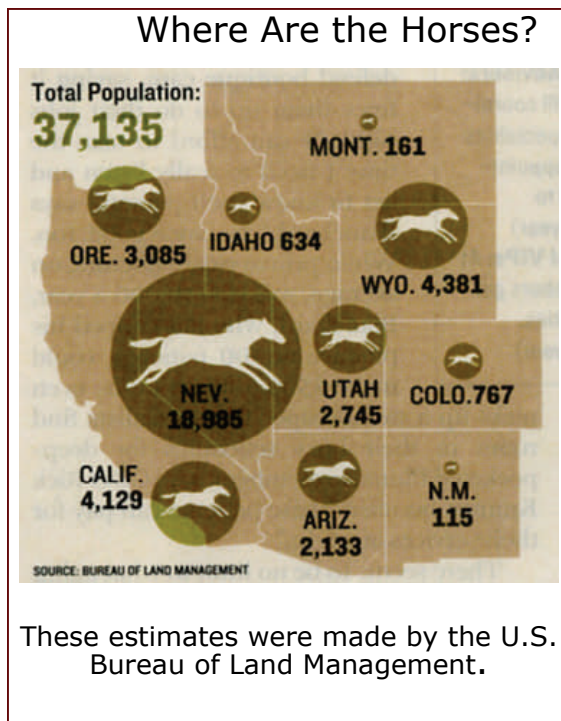
## Activity 15

# Estimating Animal Populations

Materials: identical beads (a few hundred of one color and a smaller number of another color), container for mixing beads, small scoop (optional)

### Introduction:

In 1900, there were about 2 million wild horses, or *mustangs*, in the western United States. By 1950, there were fewer than 100,000, and at the start of the 21st century only about 37,000 remain. Many horses end up in pet foods or shipped to other countries where people eat horsemeat. Also, cattle ranchers object to wild horses competing with cattle for grazing lands. But groups like the Humane Society are concerned about the decreasing numbers of horses.



Various public and private organizations keep track of the numbers of horses, turtles, moose, and other wild animals. But how are animal populations counted? It would be almost impossible to actually count the 18,985 horses that were estimated to be in Nevada in 2005. Instead, a technique called *capture-recapture* (or *mark-recapture*) is used.

A number of animals are captured and marked in some way. Fur can be trimmed, toes clipped, and collars are sometimes placed on animals. Horses are often branded on an easily seen part

of the body. Then the marked animals are released. After they have had time to mix in with the rest of the animals in an area, a second group is captured. By finding the fraction in this group that are marked, an estimate of the entire population can be made.

For example, assume that a group of branded horses has spread out evenly throughout the horse population. Then for any size group of these horses, the fraction that is branded should always be the same. (See Figure 1.) Figure 1a represents a smaller group of the herd pictured in 1b.

- There are 4 marked horses out of a total of 12 in the smaller group.
- There are 16 marked horses out of a total of 48 in the herd.
- The ratios of  $4/12$  and  $16/48$  are both equal to  $1/3$ .

In any random collection of horses from this herd, you can expect the fraction of marked horses to be about  $1/3$ .



Figure 1a

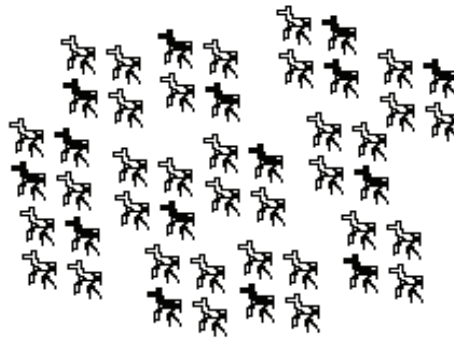


Figure 1b

### Preparation:

Work in small groups of 2 or 3.

Obtain a container of beads or other small objects that are one color.

### Explore:

#### 1. Capture and Mark

a. Use the small beads or other small objects in your container to represent a horse population. Each bead in your container represents one horse in a population to be estimated. Scoop out some of the beads and count them. How many beads did you scoop out of your container?

b. Let the beads that you scooped out represent the horses that will be branded. To simulate branding horses, replace each bead that was removed from the container with a bead of a different color. Then put the “marked” beads back in the container with the “uncaptured” beads. Mix the beads in the container well. This is equivalent to letting branded horses mix in with the population of unbranded horses. How many “marked” beads did you put back in your container?

## 2. Recapture and Use a Proportion

a. After the beads have been mixed, scoop out a second group of beads and count the number in this group.

b. How many of these beads are second-color beads that have been “recaptured”?

c. Complete Table 1 to summarize your findings so far.

First captured group	
Number captured and marked	
Total population size	Unknown
Second captured group	
Number captured	
Number that were marked	

Table 1

3. What is the ratio of the number of marked (recaptured) beads to the total number of beads in the second group?

4. If the marked beads were well mixed with the unmarked beads, any captured group should contain the same  $\frac{\text{marked beads}}{\text{total beads captured}}$  ratio as the entire population.

A statement that two ratios are equal is called a **proportion**. Complete the proportion below by comparing the ratio of branded horses to total horses captured for the second captured group and for the ratio of branded horses to total horses captured for the whole population.

$$\frac{\text{marked beads (in second captured group)}}{\text{total beads captured (in second captured group)}} = \frac{\text{marked beads (in the whole population)}}{\text{total beads (in the whole population)}}$$

$$\frac{?}{?} = \frac{?}{\text{population}}$$

Notice that since the whole population is unknown, the word “population” is written in the appropriate place in one of the ratios.

5. Since the two ratios in the proportion are equal, you can think of them as equivalent fractions. But one denominator is unknown. What number must the numerator of the known fraction be multiplied by to give the numerator of the other fraction?

6. Use this number to estimate the total number of beads in the container.

7. Repeat parts 1-6 to find a second estimate of the bead population. Is the result similar to your first estimate?

### Practice:

8. Suppose a similar procedure is used to find the number of horses in a large grassland. Twenty horses are initially captured and branded. Then they are released into the grassland. After a week, 80 horses are captured. Three are found to be branded. Write a proportion that models this situation.

9. Use your proportion from part 8 to estimate the population of horses in this grassland.

## IR Notes to the Instructor

**Objective:** Use beads or other small objects to simulate a capture/recapture technique for estimating animal populations.

- This activity is based on the Petersen method for estimating animal populations. C. G. J. Petersen was a Danish fisheries biologist who in 1896 first used mark-recapture protocols for such purposes.
- Any small identical objects can be used in place of beads, but the total number for each group should be large. White beans can be used, and then marked with a magic marker.
- Any kind of container that allows thorough mixing can be used, such as paper bags or plastic milk bottles.
- Instead of using a scoop, students can just grab a handful of beads. The number need not be consistent between the first and second sampling.
- Decimals can be used in parts 5 and 6. A calculator can also be used here without obscuring the mathematical development of solving a proportion.
- There will not necessarily be close agreement between the two estimates in parts 6 and 7, variations of 20% being common. This variation does, however, provide an opportunity for a discussion of how this kind of estimate can be improved by repetition.

The activity introduces the solution of a proportion through a relatively seamless connection to students' previous work with equivalent fractions. More efficient algorithms may be introduced later.