Chapter 5 Lesson 2 of 5

Section(s): 5.7 Solving Quadratic Equations by Factoring

Desired Understandings

- Be able to identify solutions to factored quadratic equations using the Zero Product Principle (ZPP)
    - Must have 0 on one side of the equation – may require manipulation
    - Be able to give factorization if solutions to the equation are given
- Know how to identify the solutions of a quadratic equation by graphing
    - Understand why you will always have two factors (if factorable) but could have only 1 solution
    - Be aware that a quadratic equation may not have a solution and why (parabola w/o *x*-intercepts, polynomial not factorable over **R**)

Framework Questions to Emphasize:

What sort of answer should I expect from the procedure "Solve Quadratic Equation"?

How do I execute the procedure "Solve Quadratic by Factoring"?

What can I use the procedure "Solve Quadratic Equation" to do?

Why is solving by factoring an effective procedure to solve quadratic equations? When is it useful/applicable?

How can I verify solutions to quadratic equations?

Assessment

Communicate: (Framework ideas, misconceptions)

1. Why is it important that one side of the equation is zero?

2. Why doesn't a constant factored out in front contribute a solution?
3. Given the solutions to a quadratic equation, $ax^2 + bx + c = 0$, what else can you tell me about the function and its graph? *Can determine factors, intercepts*
4. What sort of answer do you expect in solving a quadratic equation? Explain.

Calculate/Compute:

1. $y(y+5) = 0$
2. $3(4x+9)(5x-4) = 0$
3. $9x(3x-2)(2x-1) = 0$
4. $y(y+8) = 16(y-1)$
5. $(t-3)^2 = 36$
6. $(t-5)^2 = 3(5-t)$
7. $1 + x^2 = 2x$
8. $2y^2 + 12y = -10$

Problems #1-3 are expected skills at the end of this lesson. #4-8 will be good examples of equations the students are expected to solve after the chapter is finished.

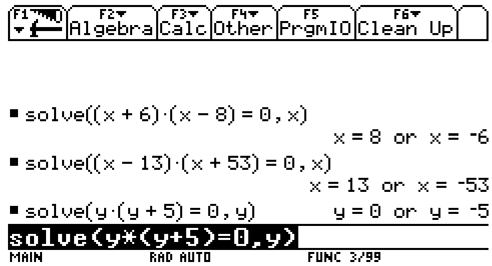
<u>Role of CAS</u>: In this lesson, the CAS will solve factored and unfactored quadratic equations, in addition to graphing the corresponding functions. This will allow for a discussion of the underlying concepts of solving a quadratic equation, such as the relationship between factors and solutions, the relationship between zeroes and intercepts, and what to expect in the process. Students are empowered by being able to solve any quadratic equation and know that the factoring procedure is now important to learn for when the calculators are taken away.

Begin Lesson

#1
Use CAS to introduce the ZPP and discuss why factoring is effective and valid to use for solving.

Have students use the solve command to solve the following problems from 5.7:

$$(x+6)(x-8)=0$$
$$(x-13)(x+53)=0$$
$$y(y+5)=0$$



Discuss their ideas of what the “black-box” procedure could be.

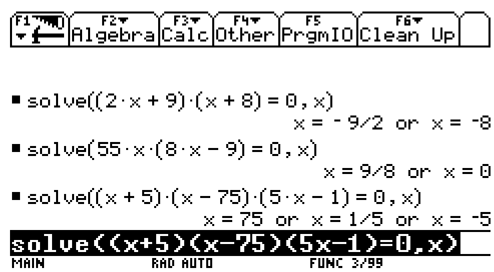
These first few examples should reveal the idea behind the Zero Product Principle.

Formally introduce the Zero Product Principle to discuss the Framework question: **Why is the procedure effective and valid?**

#2
Use CAS to look at some different situations that can occur and address common mis-conceptions

Now have them solve the following problems:

$$(2x+9)(x+8)=0$$
$$55x(8x-9)=0$$
$$(x+5)(x-75)(5x-1)=0$$



Discuss the subtle differences and common misconceptions:

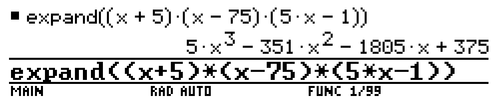
Why did we get fractions for answers to these equations? *Coefficients on the x terms in the binomial factors.*

How can we solve these mentally? *Take the opposite of the constant term divided by the coefficient of the variable term.*

Why isn’t 55 an answer to the second equation? *Doesn’t give a zero in the product. Monomial coefficients can’t give us zeroes, only substitutions for the variable.*

Why are there three answers to the third example? *Three factors.*

Use the expand command: 



What is the highest exponent on the polynomial (Degree)? *Three, called a cubic.*

We have been working with quadratics. If we multiply out the other examples above (using FOIL or distribution) we would get polynomials with powers of 2.

How many solutions do we expect from polynomials? *The same as the degree*

(Note: we will confront them with repeated factors next to further develop their understanding.)

#3
Use CAS to look at what happens to give one or no solutions by looking at factors and graphs.

These next examples aren't factored but as I write them, guess how many solutions you expect:

$$x^2 + 7x + 6 = 0$$
$$x^2 - 3x = 0$$
$$0 = 25 + x^2 + 10x$$

Two, because of the exponent.

Have the students use the solve command and, as they get the answers, **have them write down what they think the factors should be**. They will only get one answer for the last example, providing the following discussion opportunity.

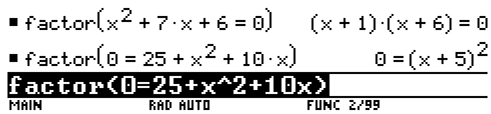
Consider: $x^2 + 7x + 6 = 0$

What did the solve command give you? Solve command gave x = -1 or x = -6

Will the polynomial factor? What did you guess?

Use CAS factor command:

Did you get it right?



Now think of: $0 = 25 + x^2 + 10x$

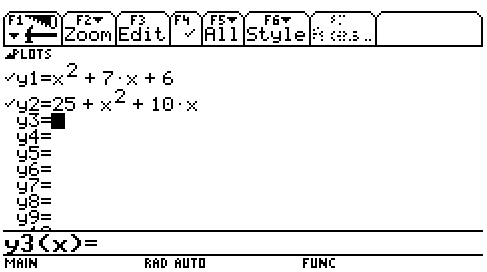
Solve command gave x = -5.   Why only one answer?

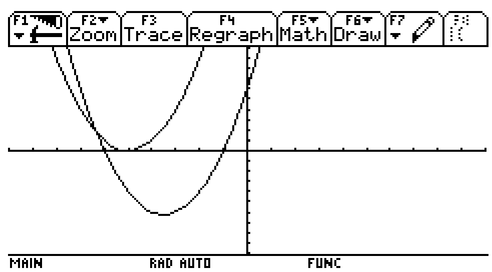
Use the factor command (see above screen)

Still get the expected 2 factors because $(x+5)^2 = (x+5)(x+5)$, but they are the same so we only got one solution (zero times zero is zero just the same).

#4
Use CAS to look at relationship between solutions and intercepts and the different number of solutions to expect 0, 1, or 2.

Let's graph the above examples:





(In zoom standard <F2>:6)

Find the points where x = -1 and -6 on the first graph and x = -5 on second graph.

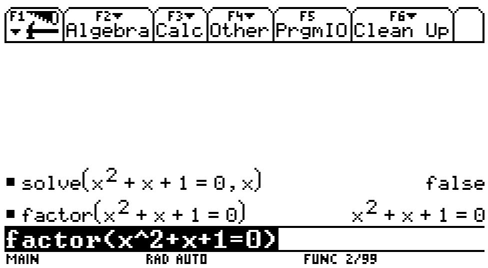
Notice these points are the intercepts. The picture of the second graph also shows why it only has one solution – it just touches the graph and doesn't cross twice.

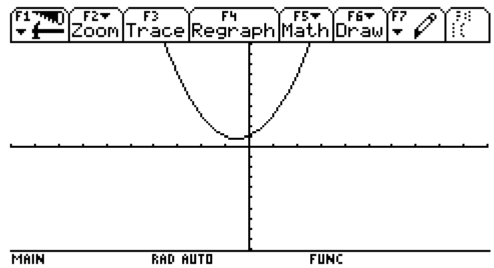
Optional
Not a focus of the text or class to discuss no solution. They will not have these problems in HW or test.

Point out that if a graph was completely over or under the axis it would have no solutions.

What would this mean for factoring?

Check with the example $x^2 + x + 1 = 0$. Calc won't solve (returns false) or factor and the graph illustrates why – there are no intercepts.





#5
Concluding framework discussion about the results we've seen and to reinforce FW objectives.

Framework discussion:

1. What sort of answer should I expect when solving a quadratic equation? *0,1, or 2.*

In our case 1 or 2 but always has 2 factors even if they are the same.

2. How do I solve a quadratic equation? *Get zero one on side and then:*

Factor and use ZPP, or Graph and find intercepts.

3. Why is it important to have zero on one side? Why does this make the process work so well? *We only know what on of the factors has to be if they multiply to 0. If they multiplied to 12 then we don't know what the factors should be: 2 and 6 or 4 and 3, etc.*

4. How can I verify my answer? *Plug in and compute to verify the equation is true, graph and look at the intercepts, write the factors based on your answers and multiply (reverse factor).*

5. What can I use solving quadratic equations to do?

- *If not already known, I can find its factorization and the intercepts of its graph.*
- *I can solve any equation that is a product on one side and zero on the other.*
- *I can solve certain applications like we did last time: areas, consecutive integers, Pythagorean formula triangle problems*

IF TIME
Work through an application problem, reinforce zero needed on one side and recall an example from the last lesson.

The length of a rectangle is 2 ft more than the width. The area of the rectangle is 48 ft^2. Find the length and width.

Set up the above problem. This problem requires multiplying the factors and then subtracting the 48. This reinforces that you must have zero on one side and that it may take some work to get there. Then use the CAS to factor (not solve). The students can then read off the answers from the factored form.

Do other examples from 5.8 if time allows.