Using Computer Algebra Systems in a Developmental Algebra Course

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Curriculum Design Considerations

- Procedural Understanding Framework (FW)
- Technology Use
- Classroom Environment and Teacher/Student Interaction

Framework for Procedural Understanding

- Designed to help students develop a deeper, well-connected knowledge of mathematics:

1. (a) What is the goal of the procedure, and (b) what sort of answer should I expect?
2. (a) How do I execute the procedure, and (b) what are some other procedures I could use instead?
3. Why is the procedure effective and valid?

Framework for Procedural Understanding

4. What connections or contextual features could I use to verify my answer?
5. When is this the "best" procedure to use?
6. What can I use this procedure to do?

(Hasenbank, 2006, p. 7-8)

Computer Algebra Systems (CAS)

- Software or calculator able to perform numerical, graphical, and symbolic manipulations
- Compensation tool for developmental students
- Allows students and teachers to focus on conceptual understanding over manipulation skills

Roles of CAS

1. Trivialization: Empowers students to handle more complex and realistic problems that would be difficult to do by hand
2. Experimentation & Generalization: Students rapidly perform procedures and accurately generate examples
3. Visualization: Illustrates graphic, numeric, and algebraic representations
4. Concentration: Students focus on higher-level skills and concepts to be learned. Students make the executive decisions of how to model the problem.

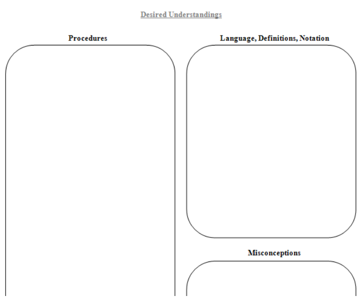
Discourse in Mathematics

- Classroom discourse behaviors
  - Questioning
  - Explaining mathematical thinking
  - Source of mathematical ideas
  - Responsibility for learning (Hufferd-Ackles et al., 2004)
- Sfard's communicational approach to cognition suggests a learning theory for the framework curriculum
- Curriculum structured to emphasize classroom discourse

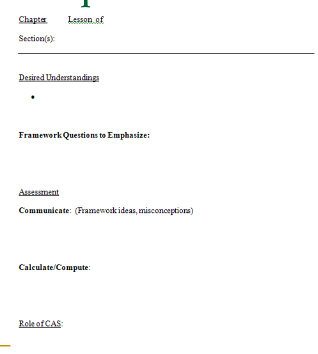
Backwards Curriculum Design
(Wiggins and McTighe)

1. Identify Desired Understandings
   - List all procedures, notation, language, misconceptions
   - Framework questions for each main procedure
   - Uses of CAS to answer framework questions and address symbolism and misconceptions
2. Determine Acceptable Evidence
   - Procedural skill
   - Framework understanding
3. Plan Instructional Activities
   - Reordering the sections
   - Activities using CAS to address framework

Design Templates



Design Templates



Chapter Lesson of

Section(s):

Desired Understandings

-

Framework Questions to Emphasize:

Assessment

Communicate: (Framework ideas, misconceptions)

Calculate/Compute:

Role of CAS:

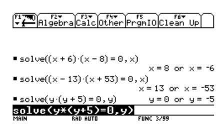
Activity Goals

- Generate data quickly and accurately
  - Students can:
    - Explore
    - Observe patterns
    - Make and test conjectures
- Create mathematical representations
  - Symbolic
  - Numeric
  - Graphical

Sample Activity

Have students use the solve command to solve the following problems from 5.7:

$(x+6)(x-8)=0$
$(x-13)(x+53)=0$
$y(y+5)=0$



Discuss their ideas of what the "black-box" procedure could be.

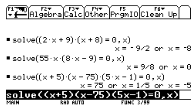
These first few examples should reveal the idea behind the Zero Product Principle.

Formally introduce the Zero Product Principle to discuss the Framework question: **Why is the procedure effective and valid?**

Sample Activity

Now have them solve the following problems:

$(2x+9)(x+8)=0$
$55x(8x-9)=0$
$(x+5)(x-75)(5x-1)=0$

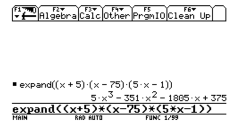


Discuss the subtle differences and common misconceptions:

Sample Activity

Why are there three answers to the third example? *Three factors.*

Use the expand command:



What is the exponent on the polynomial? *Three, called a cubic.*

Sample Activity

These next examples aren't factored but as I write them, guess how many solutions you expect:

$x^2+7x+6=0$
$x^2-3x=0$ *Two, because of the exponent.*
$0=25+x^2+10x$

Have the students use the solve command and as they get the answers, **have them write down what they think the factors should be**. They will only get one answer for the last example, providing the following discussion opportunity.

Sample Activity

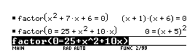
Consider: $x^2+7x+6=0$

What did the solve command give you? Solve command gave $x=-1$ or $x=-6$

Will the polynomial factor? What did you guess?

Use CAS factor command:

Did you get it right?



Now think of: $0=25+x^2+10x$

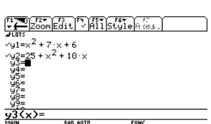
Solve command gave $x=-5$. Why only one answer?

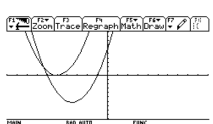
Use the factor command (see above screen)

Still get the expected 2 factors because $(x+5)^2=(x+5)(x+5)$, but they are the same so we only got one solution (zero times zero is zero just the same).

Sample Activity

Let's graph the above examples:





(In zoom standard <F2>:6)

Find the points where $x=-1$ and -6 on the first graph and $x=-5$ on second graph.

Notice these points are the intercepts. The picture of the second graph also shows why it only has one solution – it just touches the graph and doesn't cross twice.

Population and Sample

- Population: COT introductory algebra classes
  - 8 sections, 30 students maximum per section
- Instructor chosen based on CAS familiarity and willingness to support research project
- One section of this instructor's three sections was taught using the CAS/framework curriculum
  - Chosen based on pretest scores to judge equivalency of groups on algebra skill

Treatment Section

- 29 students began semester; 18 took final exam
- Monday – Thursday, 11:00 – 11:50 a.m.
- Daily calculator use
- Additional framework-based quizzes
- Researcher present during most class periods

Instructor Impressions/Observations (As gauged by interview with instructor)

- CAS effectively engaged students in lessons
- Calculators well-received; few student difficulties
- Instructor struggled to get responses from students, possibly because he was trying to get so much more discussion from the treatment class
- Time issues similar to preparing for additional course
- Syllabus requirements were achieved

Instructor Impressions/Observations

1. CAS kept students engaged in lesson. (Can be difficult task with developmental students.)
2. Students enjoyed using calculators in class and on parts of each test. Eliminated the question, "Why can't we use calculators in this class?"
3. Excellent, concept-focused lesson plans!
4. Classroom layout and equipment not optimal for simultaneous CAS projection and board work.

Instructor Impressions/Observations

5. In retrospect, treatment section seemed to be an outlier, which presented special challenges:
   - Persistence through final exam was 62%, compared to 75% and 78% in other two sections
   - Treatment section was reticent to engage in discussion compared to other two sections
   - In teaching 30 developmental math sections over the past four years, only two stand out as exceptional:
     - ✓ The "comatose" treatment section
     - ✓ A "hyperactive" section the following semester

Student Interviews

- "It was easy to learn" to use the calculators
- We used the calculators too much
- Rather learn by hand than use CAS
  - "I'd rather learn how to do everything"
  - "Rather do more problems on my own"
- Testing issues
- Philosophy of mathematics

Student Feedback to Instructor

- I expected some resistance to daily CAS use. Didn't get it. Students had their calculators in class every day, almost without exception.
- Student post-course survey:

  "Rate the course in general."
  - Treatment section score: 92% favorable
  - Scores from other two sections: 66% favorable

Observation Conclusions

- Key component of discourse was not achieved at the level intended by the curriculum
  - Lack of confidence in sharing ideas
  - More motivated to answer questions about manual steps of procedures
- Developmental students expected continuous review and correction of misconceptions

Data Collection Instruments

Do students learning through the CAS/Framework curriculum have the same level of framework-based procedural understanding as students in the traditional courses?

- Procedural Understanding Quizzes
  - Follow-up Student Interviews
- Procedural Understanding Exam

Validity and Reliability of Procedural Understanding Exam

- Procedural Understanding Quizzes
  - Pilot of framework-based writing prompts
  - Interviews
- Independent Framework Validation
- Consistency of scores over time
  - Pearson's correlation: $r = .943$
- Inter-rater reliability
  - Pearson's correlation: $r = .932$

Procedural Understanding Exam - Sample Question

Two of your friends disagree about how to reduce the rational expression $\frac{x^2-25}{x^2-10x+25}$. Fred just cancelled the x^2 terms and the 25's and wrote $\frac{1}{10x}$. Marcie thinks Fred's answer is incorrect but is having trouble explaining it to Fred. Describe the general steps for simplifying a rational expression and why it works that way.

Comparison of Procedural Understanding

- Questions scored using 4-level rubric
- Scores on eleven questions compared:
  - Treatment Group: $M = 1.12$, $n = 16$
  - Control Group: $M = 1.10$, $n = 119$
- Independent samples *t*-test
  - $t_{133} = -.162$, $p = .871$
- ANCOVA, effect of pretest score
  - $F(1, 128) = .018$, $p = .893$

Evaluating Traditional Skill Level

Do students learning through CAS/ Framework curriculum have same level of procedural skill as students in traditional courses?

- Final exam
- Algebra skills pretest

Comparison on Procedural Skill

- ANCOVA, compare final exam scores accounting for differences on pretest
  - Treatment: *M* = 21.67, *SD* = 5.401, *n* = 18
  - Control: *M* = 23.87, *SD* = 6.250, *n* = 119
  - $F(1, 134) = 2.636$, $p = .107$

Comparison on Procedural Skill

- Gain Score Analysis, compare differences in correct answers on subset of 13 common questions
  - Treatment: *M* = 3.00, *SD* = 2.849, *n* = 18
  - Control: *M* = 2.95, *SD* = 2.537, *n* = 119
- Independent Samples *t*-test
  - $t_{135} = -.077$, $p = .938$

Conclusions - Implementation

- Learning how to use CAS was not an obstacle for developmental students
- Instructor and students were able to use CAS in the classroom without major difficulty
- Teacher preparation time was equivalent to preparing for an additional course
- Department syllabus requirements were met

Conclusions – Procedural Skill and Procedural Understanding

- Using CAS with the Framework approach was just as effective in developing procedural skill as the traditional approach
- Considerations for using the CAS/Framework curriculum to achieve deeper procedural understanding
  - Prepare for challenges of creating discussion
  - Counter student's philosophy of mathematics
  - Place emphasis on FW in assessing course grade

Lasting Changes for Instructor

- Working with CAS/Framework lessons for a semester helped to develop my inquiry skills.
- I have a better appreciation of the value classroom discussion in developing conceptual understanding.
- I continue to practice the inquiry skills I developed during this project.

Questions/Discussion