1.3 Systems of Linear Equations

Definition: A *linear equation* in variables $x_1, x_2, \ldots, x_n$ is an equation of the form $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$.
A collection of such equations is called a *system*:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m. \end{aligned}$$

A *solution* to this system is a set of values for $x_1, x_2, \ldots x_n$ such that each equation is satisfied.

Definition: Two systems of equations are *equivalent* if they have exactly the same solution set. A system of linear equations is *dependent* if one equation in the system is a linear combination of the equations.

Note: Solutions to systems can be given *parametrically*.

Definition: If you eliminate dependent equations from a system of equations, the number of equations is called the *rank* of the system.