

### 3.4 Inverses

**Note:** Suppose  $T$  is a one-to-one and onto transformation from vector space  $\mathcal{V}$  to vector space  $\mathcal{W}$ . Then we can define one-to-one and onto transformation  $T^{-1}(w) = v$  whenever  $T(v) = w$ .  $T^{-1}$  is called the inverse of  $T$ .

**Note:** We can find  $A^{-1}$  for a  $n \times n$  matrix  $A$  as follows:

- (a) Create the “double matrix”  $[A|I]$ .
- (b) Row reduce  $A$  down to  $I$  (if possible) while simultaneously performing the same operations on the right side of the “double matrix”. The  $I$  on the RHS will be transformed into  $A^{-1}$ :  $[I|A^{-1}]$ .

**Theorem 1** *Let  $A$  be an invertible matrix. Then  $AA^{-1} = A^{-1}A = I$*

**Theorem 2** *Let  $A$  be  $n \times n$ . If either one of the following hold, then  $A$  is invertible and  $B = A^{-1}$ :*

- (a) *There is a matrix  $B$  such that  $AB = I$ ;*
- (b) *There is a matrix  $B$  such that  $BA = I$ .*