The kernel method and area of directed lattice paths

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We solve enumeration and asymptotics of the area below directed lattice paths (walks on N, with a finite set of jumps). It is a nice surprise (obtained via the "kernel method") that the generating functions of the moments of the area are algebraic functions, expressible as symmetric functions in terms of the roots of the kernel. (The kernel method is a way to solve a single functional equation involving a lot of unknowns like F(z,u,q). Binding the variables z and u together gives additionnal functional equations, and some additional tricks allow to get closed form solutions to the initial functional equation).

For a large class of walks, we give full asymptotics for the average area of excursions ("discrete" reflected Brownian bridge) and meanders ("discrete" reflected Brownian motion). We show that drift is not playing any role in the first case. We also generalise previous works related to the number of points below a path and to the area between a path and a line of rational slope.

This work builds on previous results from Banderier and Flajolet or Gittenberger. We present here new results for higher moments.