A Lattice Path Interpretation of Generalized Catalan Determinants

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Abstract

The determinant of the Hankel matrix $(C_{i+j} + C_{i+j+1})_{i,j=0}^{n-1}$, where $C_m = \frac{1}{m+1}\binom{2m}{m}$, was proven to be a Fibonacci number by Cvetković et al. [**?**]. Using a bijection between non-intersecting n-routes and tilings of a rectangle with squares and dominos, Benjamin et al. generalized this result to show that the determinant of $(C_{i+j+t} + C_{i+j+t+1})_{i,j=0}^{n-1}$, is a linear combination of Fibonacci numbers, in the case where $t = 0, 1, 2$ [**?**]. Recently, Chamberland and French [**?**] generalized the last result when $t = 0$ by considering generalized Hankel matrices

$$(C_{(k-1)i+j,k} + C_{(k-1)i+j+1,k})_{i,j=0}^{n-1}$$

and

$$(C_{(k-1)i+j,k} + C_{(k-1)i+j+k-1,k})_{i,j=0}^{n-1},$$

where $C_{(k-1)n+l-1,k} = \frac{l}{kn+l}\binom{kn+l}{n}$, and showing via LU decompositions that the sequence of determinants for these matrices satisfies a Fibonacci-like linear recurrence. Here, I will present an alternate, more simplified, combinatorial proof of this result by counting non-intersecting n-routes in an appropriate digraph.

References

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