LATTICE PATH COMBINATORICS FOR MULTIPLE PRODUCT IDENTITIES

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Abstract

Lattice paths are enumerated as walks on a lattice under the Delannoy criterion of vertical, horizontal and upward diagonal steps. The Delannoy recursions are generalized for arbitrary weights in each of these directions, where the row elements of the Delannoy triangles correspond to lattice paths for all walks at a given length.

Generating functions for these sets of weighted lattice paths are derived for the individual triangles as well as for multiple triangle overlays. Using the path weight variables and overlay criteria, recursive algorithms and series expansions are used to describe the basic structure of generalized Delannoy triangles. Ordinary generating functions are expanded to yield lacunary sequences in the Delannoy row summations. The lattice path enumerations are given by generalized Chebyshev polynomials of the first and second kind.

The Jacobi triple product and Watson quintuple product identities are extended to balanced product identities. Elements of various Delannoy number triangles are derived from these multiple product identities as the coefficients in their power series expansions.

Results for lacunary Fibonacci and Lucas number sequences are highlighted as special cases for many of the multiple product identities. For the most generalized variable selection, Jacobi polynomials and hypergeometric functions are used to derive expansion terms for numerous triple and quintuple products. And an interpretation of the lattice path weights in exponential terms is shown to generate trigonometric theta functions.