

Some Graph Enumerative Problems and a lattice Paths Counting Conjecture

Shanzhen Gao

Department of Mathematical Sciences, Florida Atlantic University, Boca

Raton, FL 33431, USA

E-mail: sgao2@fau.edu

Let $t(m, n, s, t)$ be the number of graphs with vertex set $V \cup U, V \cap U = \phi, V = \{v_1, v_2, \dots, v_m\}, U = \{u_1, u_2, \dots, u_n\}$ and $degv_i = s$ for $i = 1, 2, \dots, m; degu_i = t$ for $i = 1, 2, \dots, n$, (multiple edge is allowed, and $sm = nt$). Let $f(m, n, s, t)$ be the number of bipartite simple graphs with vertex set $V \cup U, V \cap U = \phi, V = \{v_1, v_2, \dots, v_m\}, U = \{u_1, u_2, \dots, u_n\}$ and $degv_i = s$ for $i = 1, 2, \dots, m; degu_i = t$ for $i = 1, 2, \dots, n$, and no edge between v_i and u_i for $i = 1, 2, \dots, \min\{m, n\}$, ($sm = nt$). We will present some formulas for $t(m, n, s, t)$, $f(m, n, s, t)$ and discuss their relationship with lattice paths counting and a nice lattice paths counting conjecture.