Connecting the Transient Probability Functions of M/M/1/H and M/M/1

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Abstract

Traditionally, determining analytic expressions for the transient probability functions of the classical queuing systems (for example: M/M/1, M/M/1/H, M/M/2, M/M/2/H, etc.) represent different challenges for finite-state space processes in contrast to infinite-state space systems. For processes modeled as finite-state, birth-death processes, the problem may be thought of as a problem in root finding to determine the Eigen values of the Q matrix. To determine analytic solutions of the transient probability functions of infinite-state space birth-death queuing processes, an impressive range of mathematics appears in the literature including techniques in partial differential equations, transform theory, recurrence theory and combinatorics.

In this talk, we concentrate on a lattice path connection between the finite-state space and infinite-state space transient probability solutions of the same model. Specifically, we discuss a lattice path connection between the transient probability solutions of M/M/1/H and M/M/1. Since the transient probability functions of M/M/1/H are completely known in terms of explicitly known Eigen values, we show how these M/M/1/H Eigen values appear in a new solution expression for the M/M/1 transient probability functions. Our method makes use of some recently published results on solving M/M/1/H using dual processes which was presented at the 5th Lattice Path Combinatorics Conference in Greece. This latest version of the transient probability functions of M/M/1 may now be compared to previous solution forms producing some interesting combinatoric identities.

The lattice path connection between comparable finite-state space and infinite-state space models holds more generally than the single server system. This increases our interest in finding state dependent formulae for the Eigen values of common finite-state space queuing models in order to obtain solutions of the comparable infinite-state space queuing system. Examples will be discussed as time allows.