Abstract

In enumerative combinatoric, a dot-formation has many interpretation and has different forms. One typical dot-formation is $\begin{bmatrix} \bullet & \bullet & \bullet \\ & \circledast & \end{bmatrix}$. This dot-formation can be use to express steps in lattice paths, for example each dot in the top row represents an up-step, a horizontal-step, and a down-step. Also, it can be use to derive an infinite lower triangular array. For example, given the dot-formation, $\begin{pmatrix} 1 & 1 & 1 \\ & \circledast & \end{pmatrix}$, we can form our infinite lower triangular array.

$$\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 1 & 0 & 0 & 0 \\ 4 & 5 & 3 & 1 & 0 & 0 \\ 9 & 12 & 9 & 4 & 1 & 0 \\ 21 & 30 & 25 & 14 & 5 & 1 \end{array}$$

We looked at a certain type of dot-formations. These dot-formations were of infinite form.

$$\begin{pmatrix} \cdots & \cdots & \cdots \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ & \circledast & \end{pmatrix}$$

From these, we formed lower triangular arrays. These arrays were of importants because they fell under the Fundamental Theorem of Riordan Array(FTRA). Next, we found the generating function of the arrays $g(z)$ sequences and $f(z)$ sequences. From these generating functions, we notice a particular relationship between them. There relationships were given as:

$$g(z) = \left(\frac{1-z}{z}\right) f(z)$$

At first, this relationship did not give us an combinatorical interpretation, so we solved the relationship for $f(z)$. This gave us:

$$g(z)\left(\frac{z}{1-z}\right) = f(z)$$

Now, we were able to give a combinatorial interpretation. The interpretation was given the $g(z)$ sequence, we can find the $f(z)$ sequence by taking the partial sum of $g(z)$ sequence. Next, we wanted to give an interpretation of our first relationship between $g(z)$ and $f(z)$. We turned to the lattice paths referring to these sequences. Starting with the $f(z)$ sequences, we found several connections to other well-known sequences. This gave us an insight of what the connection was between our $g(z)$ and $f(z)$. This lead to a general concept to these certain type of dot-formation. And, several bijections among our g(z) sequences and these well-known sequences.