

K-trees, Catalan Identities and their Applications

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Abstract

A k -tree is constructed from a single distinguished k -cycle by repeatedly gluing other k -cycles to existing ones along an edge. If K is a nonempty subset of $\{2, 3, 4, \dots\}$, then a K -tree is obtained as above using k -cycles, where $k \in K$. In this talk, we look at the enumeration of K -trees, show that the ratio of terminal edges to total number of edges in k -trees is $\frac{k-1}{k}$, and use K -trees as models to enumerate planted plane cacti. We also use these combinatorial objects to obtain generating function identities involving generalizations of Catalan Numbers, Central Binomial Numbers, and Fine Numbers and show that the ratio of generalized Fine numbers to Catalan numbers is asymptotic to $\frac{2k}{(k+1)^2}$. The use of k -trees to generalize the Motzkin numbers and their connection to Baxter's generalization of Temperley-Lieb operators will also be discussed depending on availability of time.