

Chapter 3

2 dimensional motion

Conceptual Question# 1, 2, 4, 7, 9

Problems# 2, 8, 9, 10, 13, 18, 20, 22, 25, 32,
38, 52, 58, 65

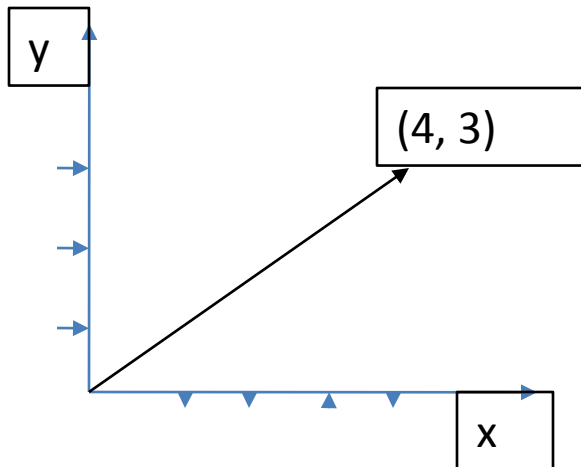
We will need to use vectors and their properties a lot for this chapter.

Notation, different ways we can write vectors

$\mathbf{A} = A_x \mathbf{x} + A_y \mathbf{y}$ unit vector notation

$\mathbf{A} = (A_x, A_y)$ vector's components

$\mathbf{A} = (r, \theta)$ magnitude and direction



$$\mathbf{A} = 4\mathbf{x} + 3\mathbf{y}$$

$$\mathbf{A} = (4, 3)$$

$$\mathbf{A} = (5, 37^\circ)$$

Magnitude of a vector

The magnitude represents 'how big' the vector is.

$$|\mathbf{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\mathbf{A} = (4, 3)$$

$$|\mathbf{A}| = \sqrt{4^2 + 3^2} = 5$$

$$\mathbf{B} = (10, 30^\circ)$$

Remember from trig that: $B_x = 10 \cos 30 = 8.6$

$$B_y = 10 \sin 30 = 5$$

Check $\sqrt{8.6^2 + 5^2} = 10$

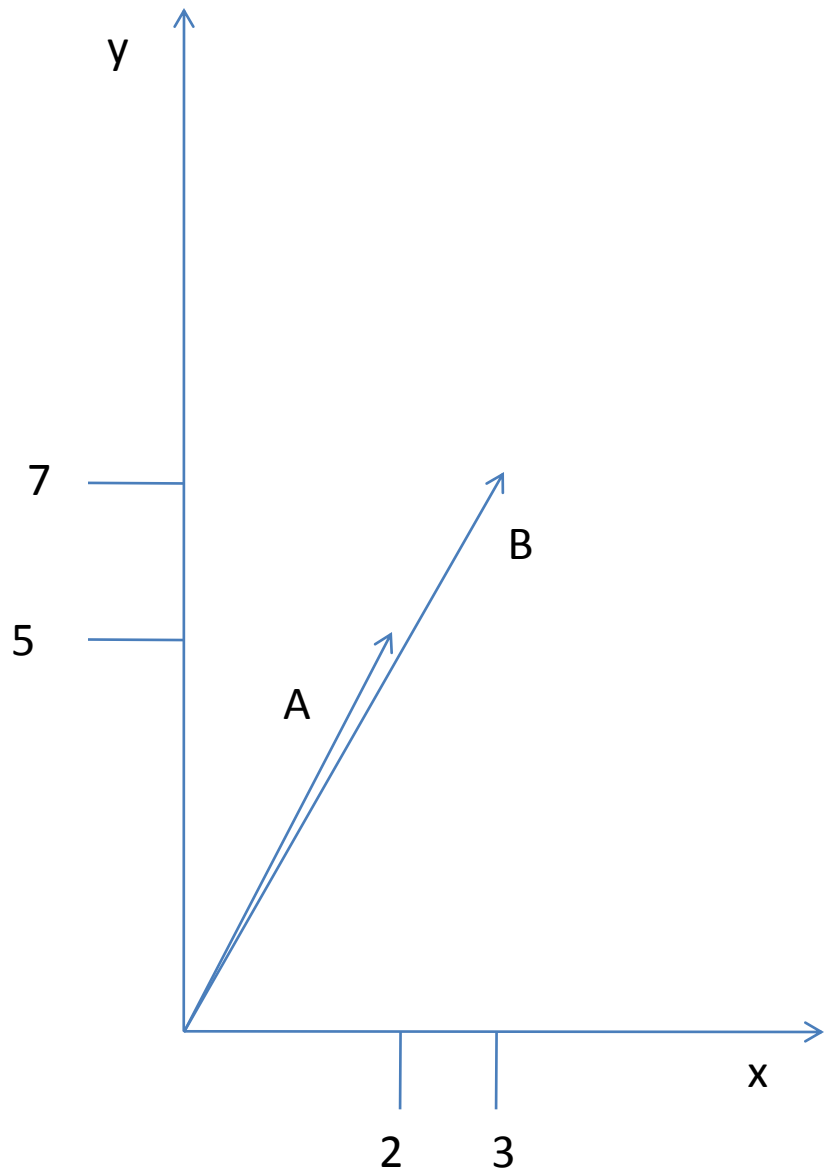
Vector properties:

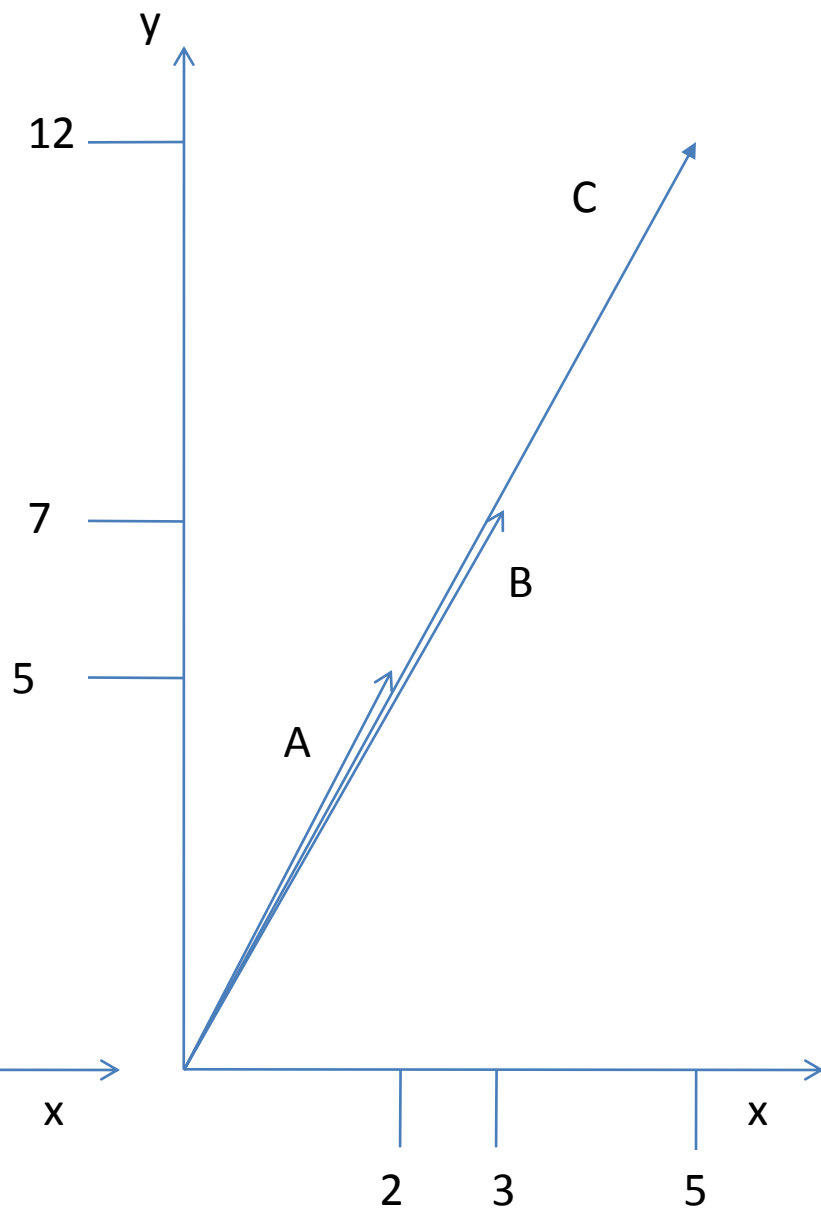
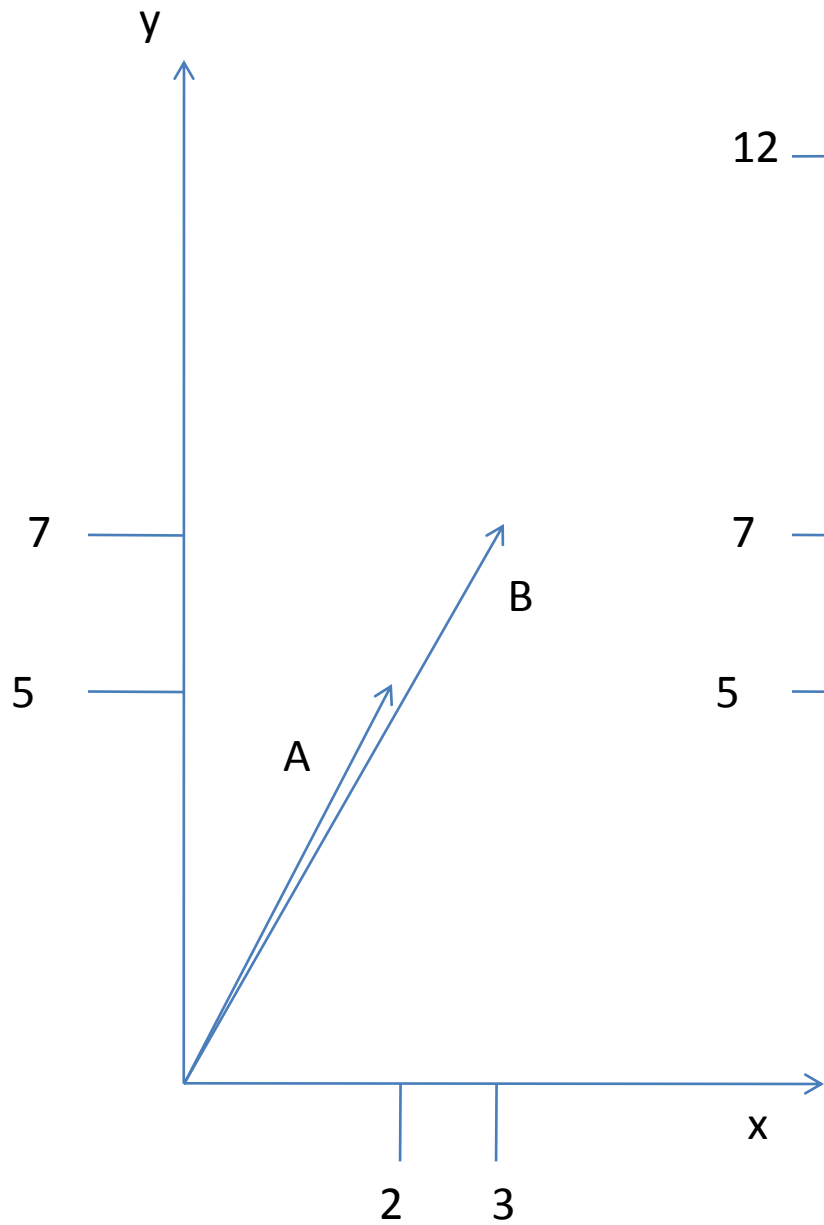
- Equality of vectors, the 2 vectors have the same magnitude and direction
- Adding vectors, can only add the same units
 - Add them up geometrically(pictures)
 - Add them up algebraically(math)

Will add these vectors: **$A + B = C$**

$$\mathbf{A} = (2, 5) \quad \mathbf{B} = (3, 7)$$

$$\mathbf{C} = (2+3, 5+7) = (5, 12)$$





Our new vector was $\mathbf{C} = (5, 12)$

It has a magnitude of $|\mathbf{C}| = \sqrt{5^2 + 12^2} = 13$

Let's find the angle that \mathbf{C} makes with the x-axis.

$$\tan \theta = 12/5$$

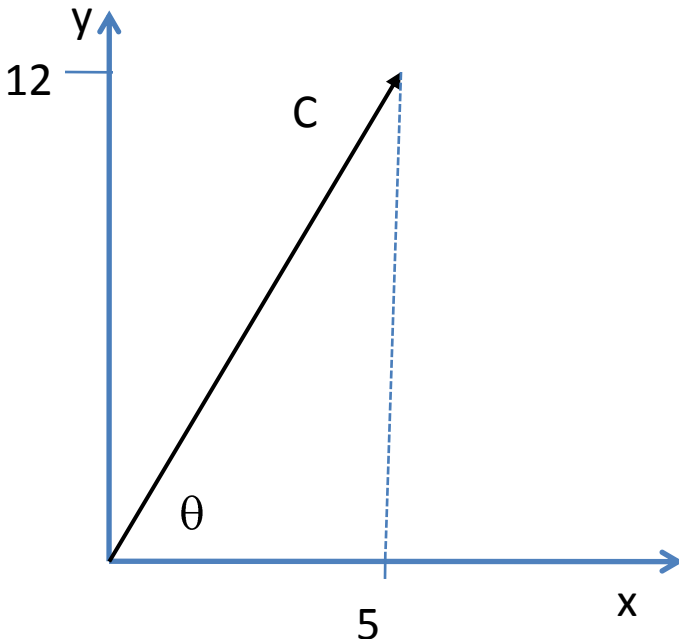
$$\tan^{-1} (12/5) = \theta$$

$$\theta = 67 \text{ degrees}$$

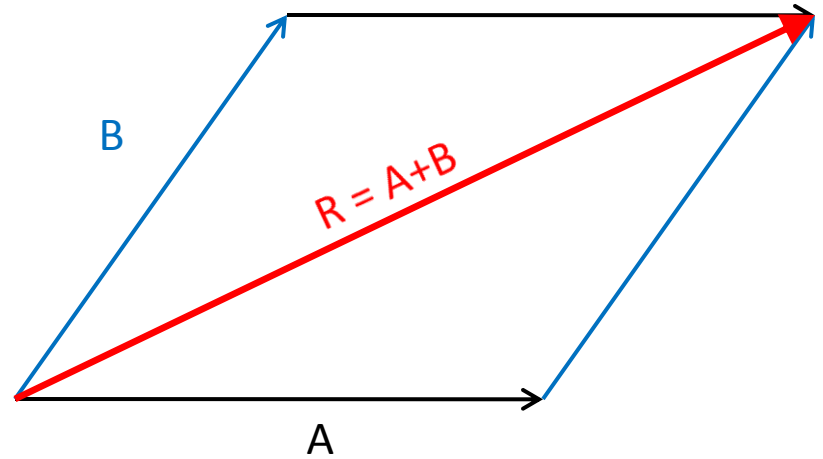
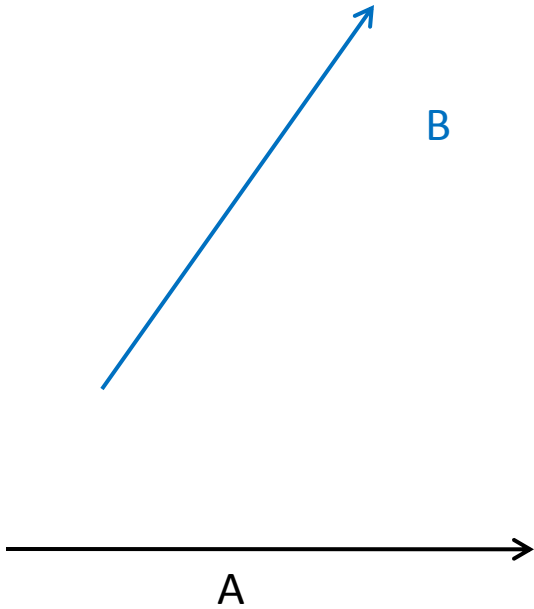
Could also do:

$$\sin^{-1} (12/13) = \theta$$

$$\cos^{-1} (5/13) = \theta$$

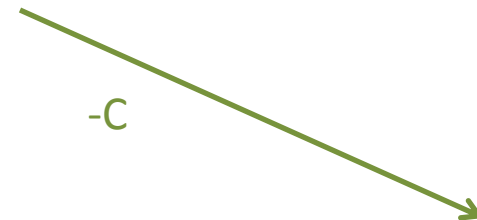
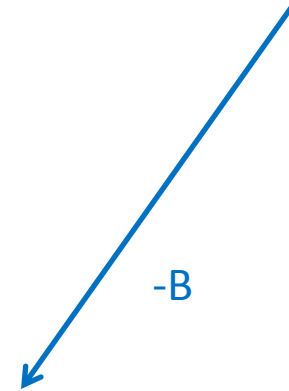
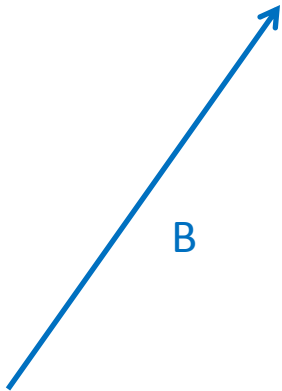
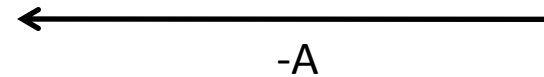
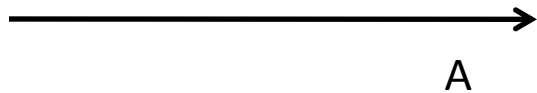


Another Example of Adding Vectors



Negative of a Vector

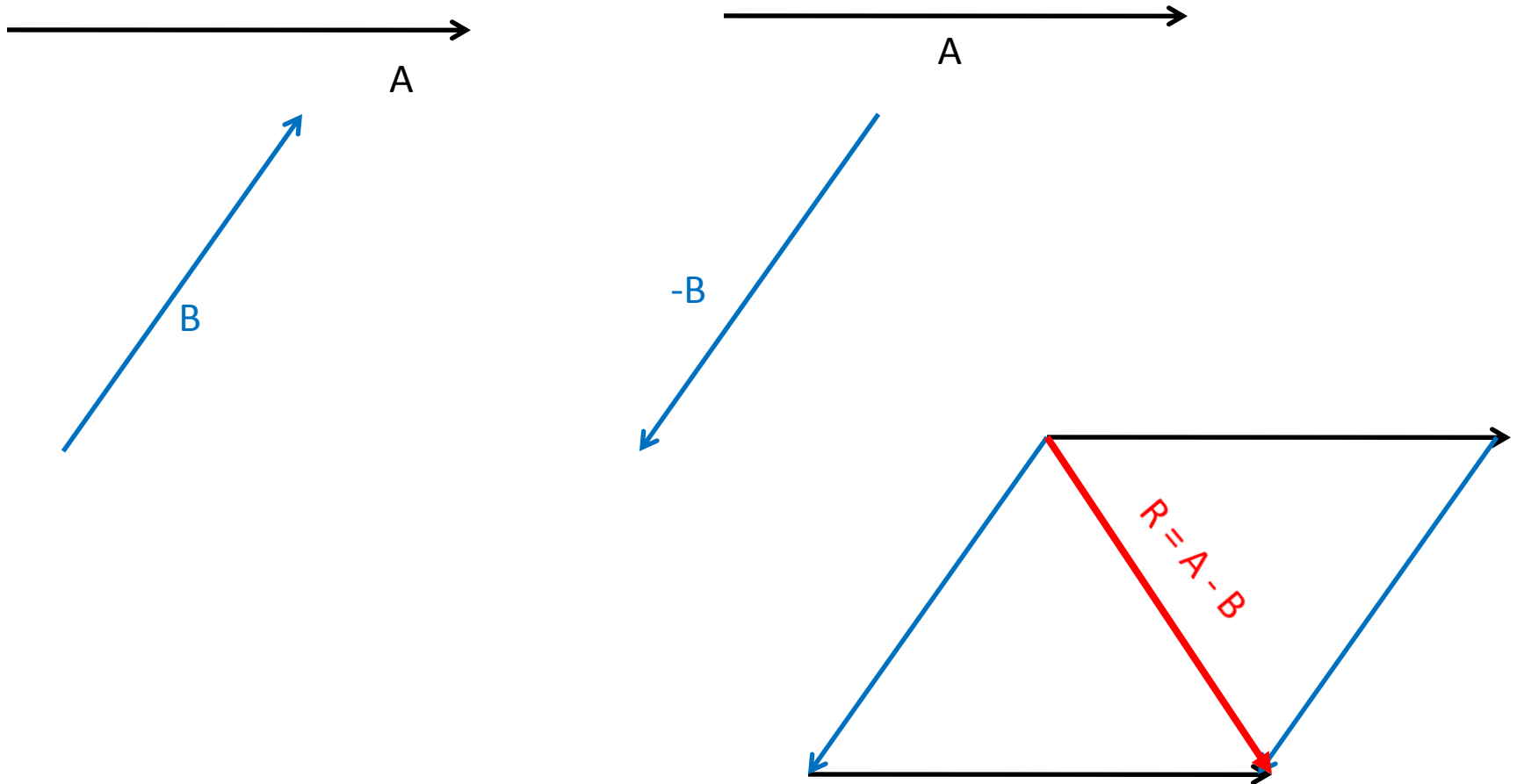
- When you have a minus sign in front of a vector, all you have to do is flip the vector to the opposite direction.



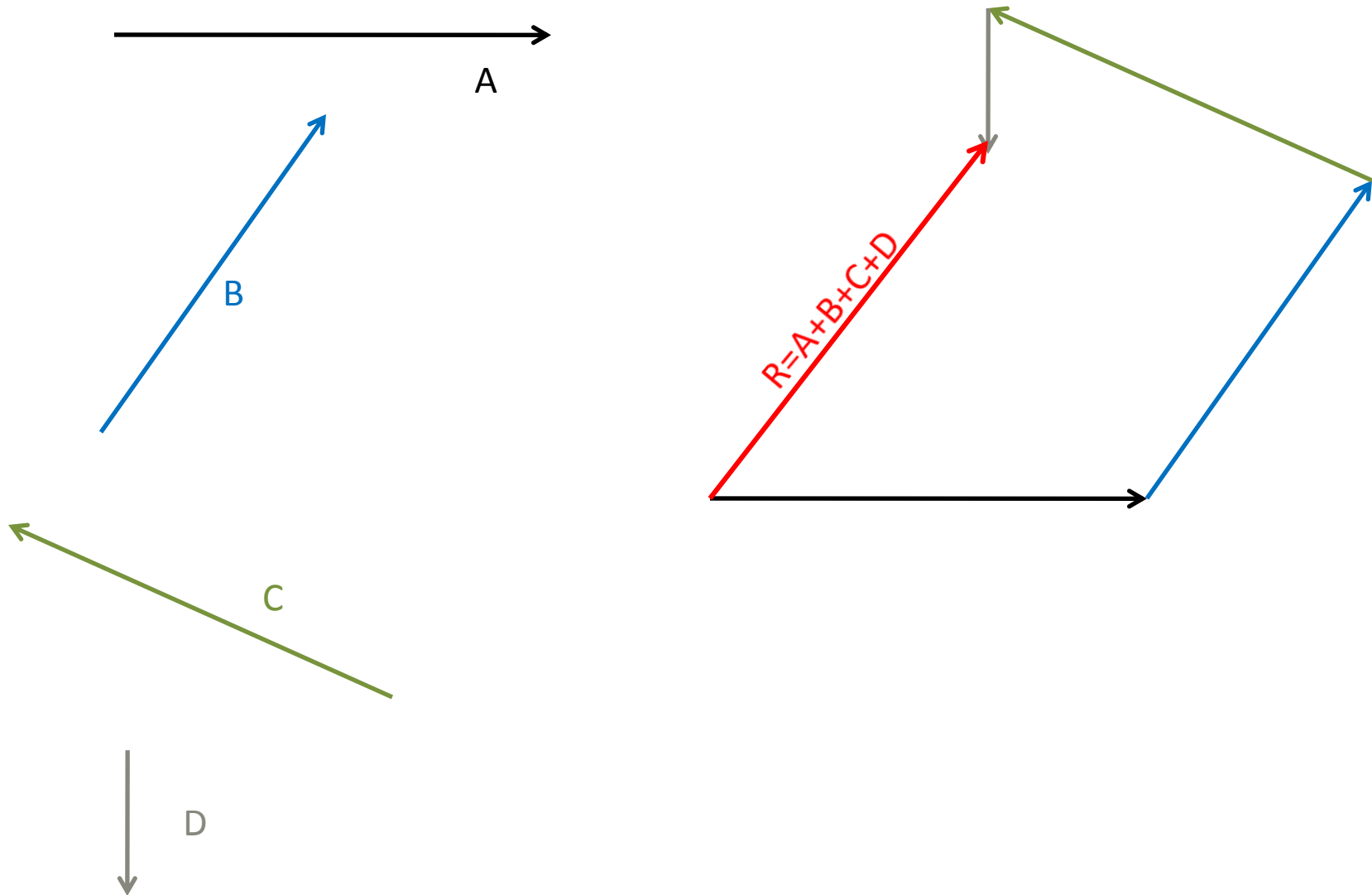
Subtracting Vectors

When doing $A - B$, think of it as adding A to Negative B .

$$A - B = A + (-B)$$



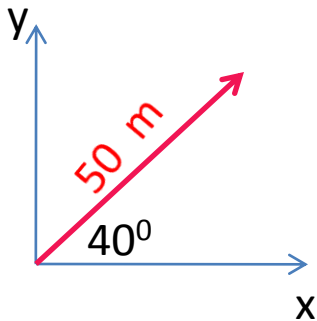
Can Add More than 2 vectors.

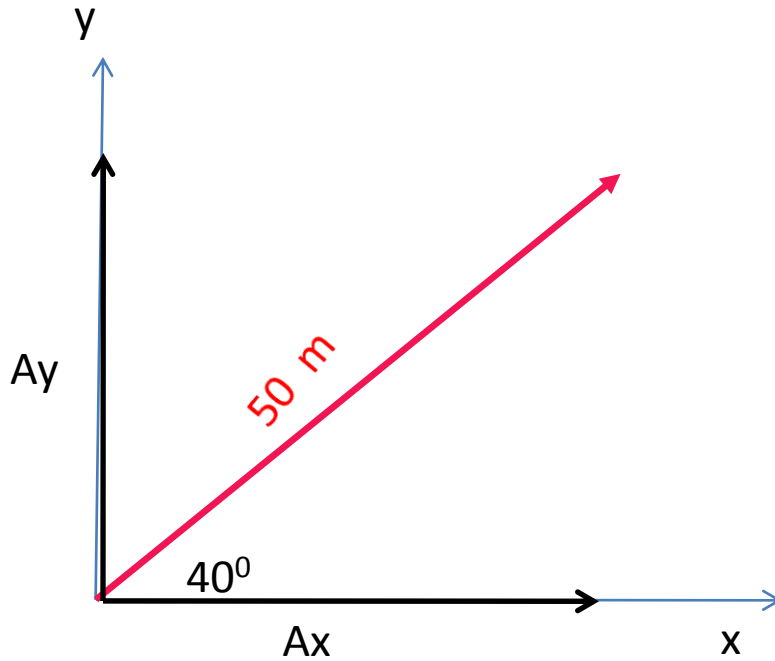


Using Vector Components

- $A_x = A \cos(\theta)$
- $A_y = A \sin(\theta)$

Find vector components of your displacement if you traveled 50 meters, in a direction 40 degrees north of east.





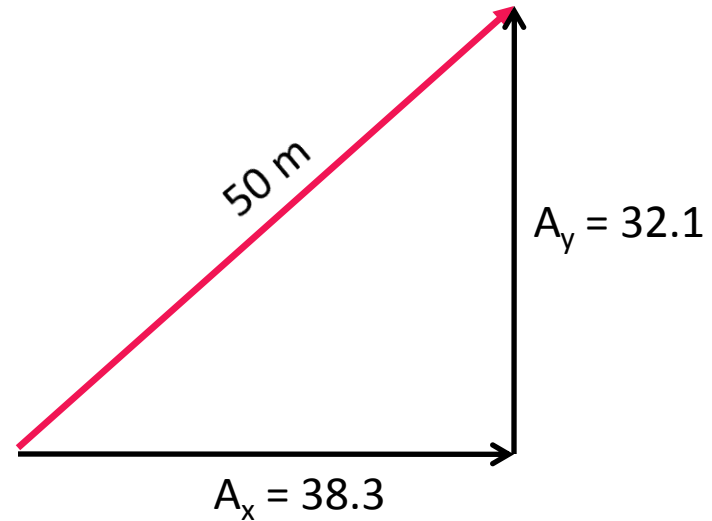
X- component = $50 \text{ m} \cos (40) = 38.3 \text{ m}$

Y- component = $50 \text{ m} \sin (40) = 32.1 \text{ m}$

As a check, let's look at a right triangle formed by the vector (50 m, 40°) and its components.

Pythagorean theorem:

Does $\sqrt{A_x^2 + A_y^2} = 50m$?



$\sqrt{38.3^2 + 32.1^2} = 49.97m$, off a little due to rounding

Also look at $\sin(40)$, $\sin(40) = 0.64$

$$\sin(\theta) = \text{opp/hyp} = 32.1/50 = 0.642$$

Another good sign that we did it correctly.

- First you hike 100 m north
- Then hike 50 m west
- Finally hike 100 meters 45 degrees northeast

Find your final total displacement using algebra and by geometry (drawing pictures).

Finding displacement using math.

- First you hike 100 m north (A)
- Then hike 50 m west (B)
- Finally hike 100 meters 45 degrees northeast (C)

First find all the components of the vectors.

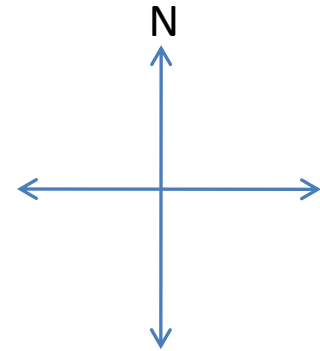
$$\mathbf{A} = (0 \text{ m}, 100 \text{ m}) \quad \mathbf{B} = (-50 \text{ m}, 0 \text{ m})$$

C has both, x and y components

$$C_x = 100 \text{ m} \cos(45) = 71 \text{ m}$$

$$C_y = 100 \text{ m} \sin(45) = 71 \text{ m}$$

$$\mathbf{C} = (71 \text{ m}, 71 \text{ m})$$



Finding displacement using math.

Add up the vector components

$$R_x = A_x + B_x + C_x = 0 \text{ m} + (-50 \text{ m}) + 71 \text{ m} = 21 \text{ m}$$

$$R_y = A_y + B_y + C_y = 100 \text{ m} + 0 \text{ m} + 71 \text{ m} = 171 \text{ m}$$

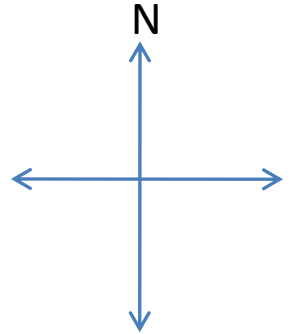
The resultant displacement vector is (21 m, 171 m)

The magnitude is then your total displacement.

$$|\mathbf{R}| = \sqrt{21^2 + 171^2} = 172 \text{ m}$$

Finding displacement using geometry (pictures)

- First you hike 100 m north (A)



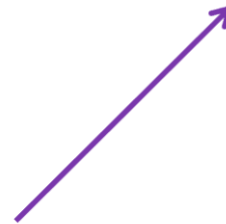
- Then hike 50 m west

(B)

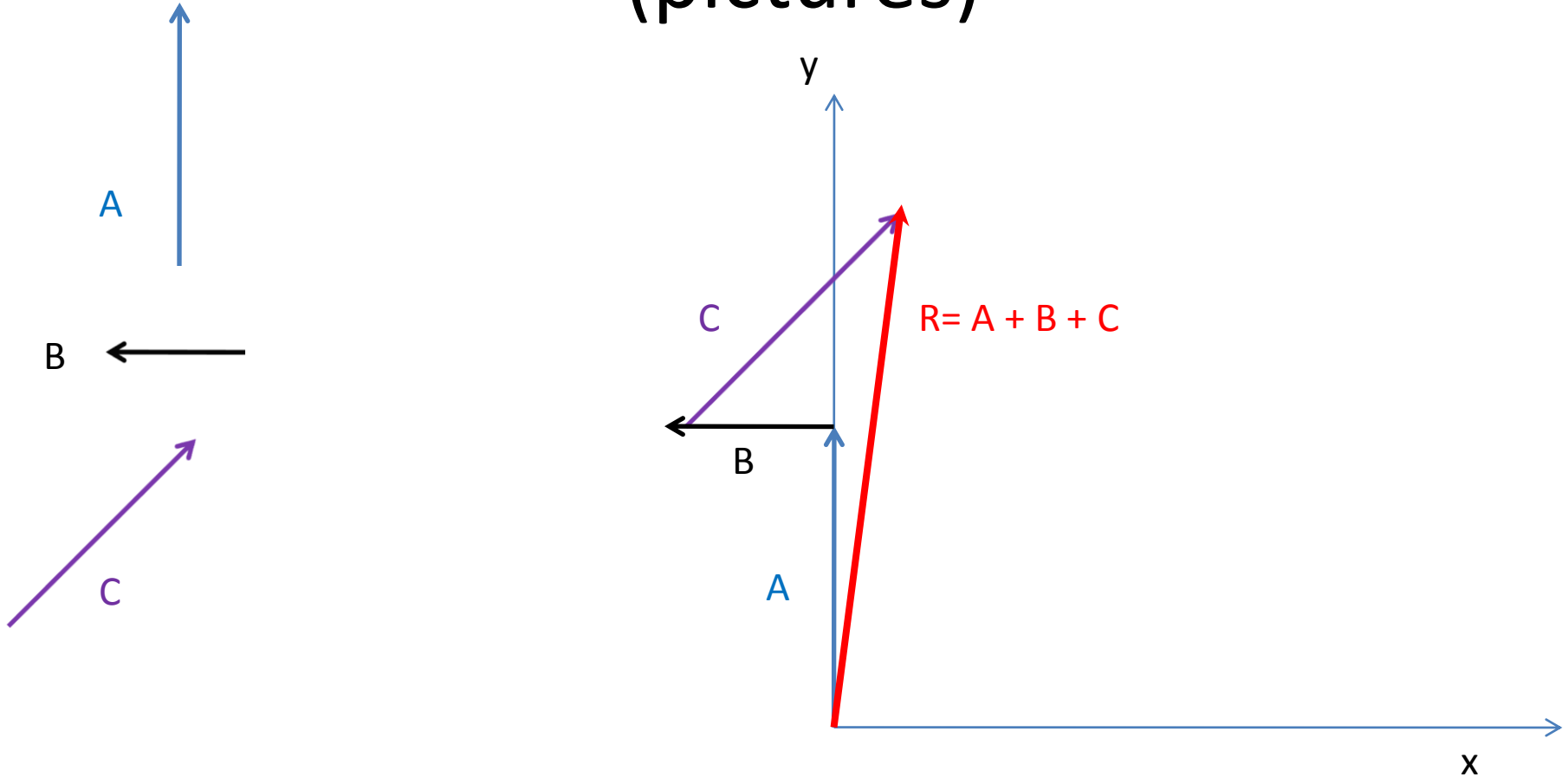


- Finally hike 100 meters 45 degrees east of North

(C)



Finding displacement using geometry (pictures)



IF everything was drawn to scale , we could measure the length of vector **R**. This will tell us the total displacement.

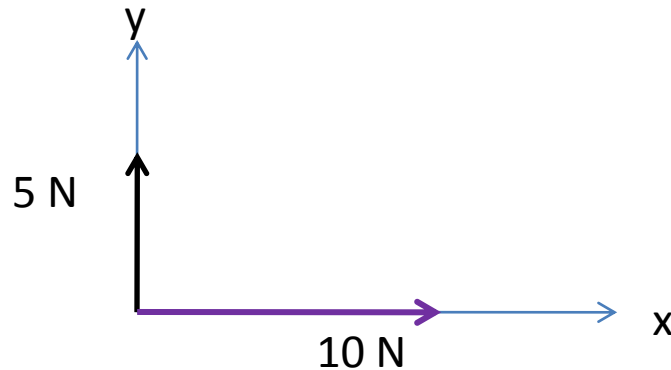
Example similar to #3.20

Two people are pulling on ropes tied to a sack of potatoes. One of them is pulling with a force of 10 newtons on the x-direction, while the other pulls with a force of 5 newtons in the y-direction. With what force does a third person need to pull in order for the sack to receive no *net* forces?

Example similar to #3.20

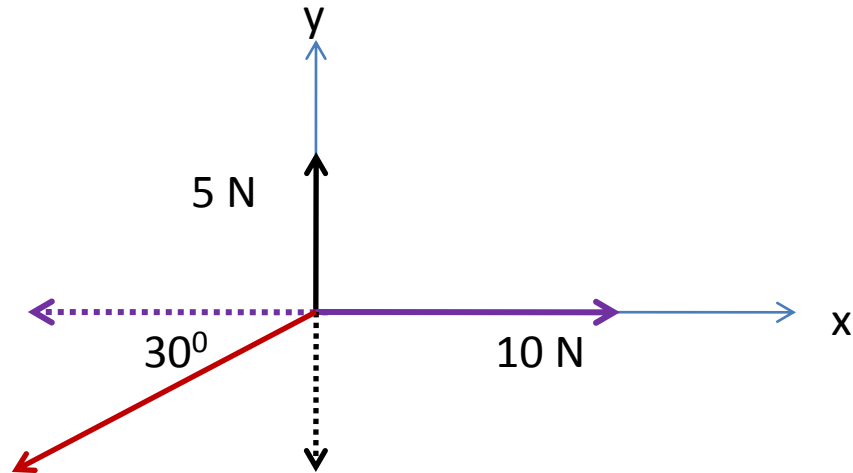
Two people are pulling on ropes tied to a sack of potatoes. One of them is pulling with a force of 10 newtons on the x-direction, while the other pulls with a force of 5 newtons in the y-direction. With what force does a third person need to pull in order for the sack to receive no net forces?

We want the 3 forces (vectors to add up to zero)



What additional vector when added make a total of zero?

Example similar to #3.20



This 3rd vector will produce a zero net force.

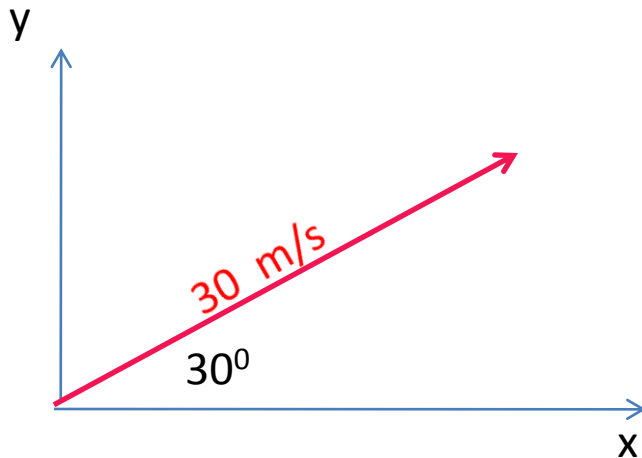
It has a magnitude of $\sqrt{10^2 + 5^2} = 11.2 \text{ N}$.

And is located at an angle of 210 degrees with respect to the positive x-axis. Or 30 degrees south of the negative x-axis

We just saw displacements and force vectors in 2-dimensions, we can likewise apply the same rules to velocity and acceleration.

The key is to break the velocity or acceleration into its vector components.

Example: A car drives 30° north of east with a speed of 30 m/s.

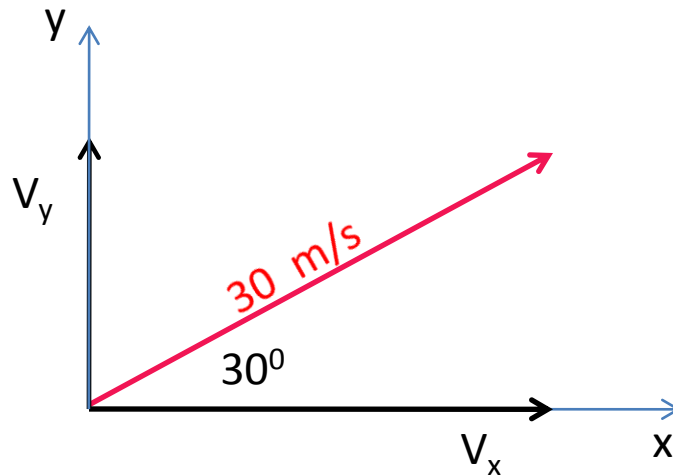


The velocity has an x-component of:

$$V_x = V \cos(\theta) = 30 \text{ m/s} \cos(30) = 26 \text{ m/s}$$

And a y-component of:

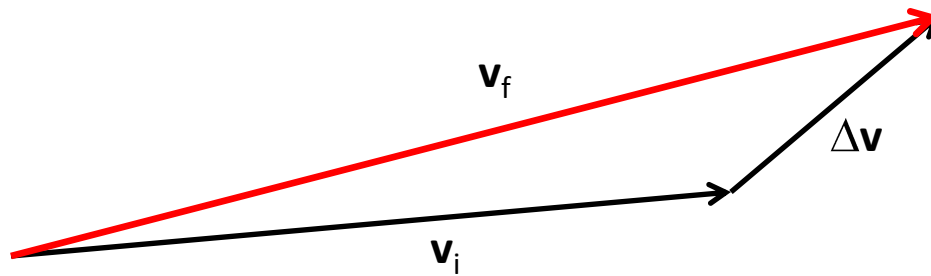
$$V_y = V \sin(\theta) = 30 \text{ m/s} \sin(30) = 15 \text{ m/s}$$



Remember the notation $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$

Since both \mathbf{v}_f and \mathbf{v}_i are vectors, so is $\Delta\mathbf{v}$.

We can rewrite the above line as $\mathbf{v}_f = \mathbf{v}_i + \Delta\mathbf{v}$



If we were to take the vector $\Delta\mathbf{v}$, and divide it by a time interval, we would have an acceleration.

Remember: $\mathbf{a} = \Delta\mathbf{v} / \Delta t$ so $\Delta\mathbf{v} = \mathbf{a} \Delta t$

Here are some more examples

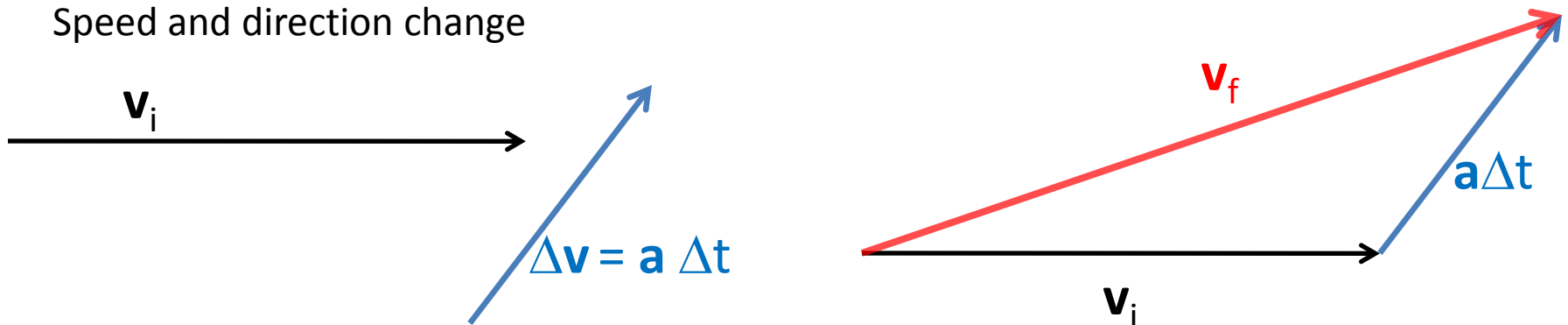
Object speeding up



Speed is decreased

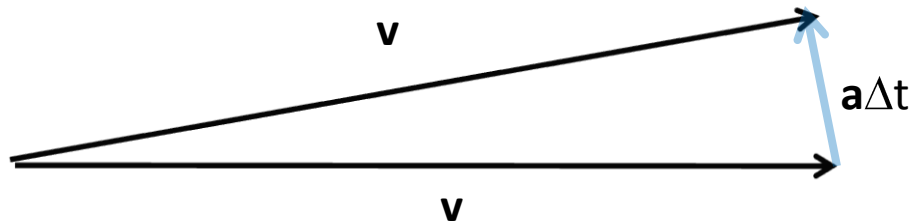


Speed and direction change



Here's an example where the direction changes but the magnitude stays the same.

$$\mathbf{v}_i = \mathbf{v}_f = \mathbf{v}$$



Acceleration

Since acceleration is a vector and can be split up into components, those components affect the similar components of the velocity.

The x-component of acceleration changes the x-component of the velocity.

The y-component of acceleration changes the y-component of the velocity.

$\mathbf{a} = \Delta\mathbf{v}/\Delta t$ (\mathbf{a} and $\Delta\mathbf{v}$ are vectors, Δt is a scalar)

$$\mathbf{a}_x = \Delta\mathbf{v}_x/\Delta t$$

$$\mathbf{a}_y = \Delta\mathbf{v}_y/\Delta t$$

$$\Delta\mathbf{v} = \mathbf{a} \Delta t$$

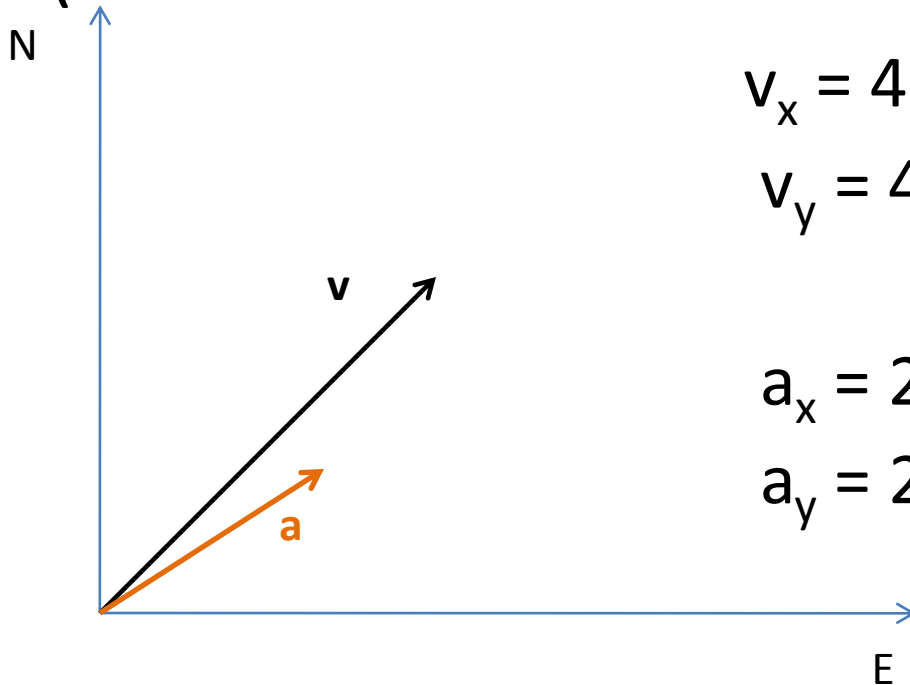
$$\Delta\mathbf{v}_x = \mathbf{a}_x \Delta t$$

$$\Delta\mathbf{v}_y = \mathbf{a}_y \Delta t$$

Example

Driving a boat 4 m/s with a heading of 45 degrees north of East. When you set up a sail, the wind accelerates the boat 2 m/s² in the direction 30 degrees north of East.

(Let's first look at these vectors)



$$v_x = 4 \text{ m/s} \cos(45) = 2.8 \text{ m/s}$$

$$v_y = 4 \text{ m/s} \sin(45) = 2.8 \text{ m/s}$$

$$a_x = 2 \text{ m/s}^2 \cos(30) = 1.7 \text{ m/s}^2$$

$$a_y = 2 \text{ m/s}^2 \sin(30) = 1 \text{ m/s}^2$$

Boat example continued.

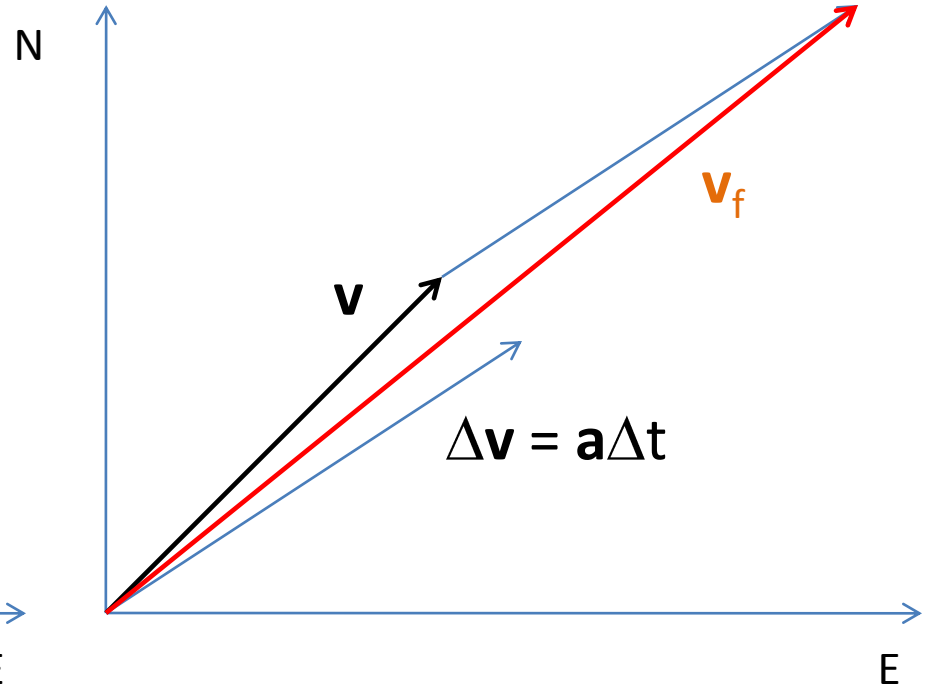
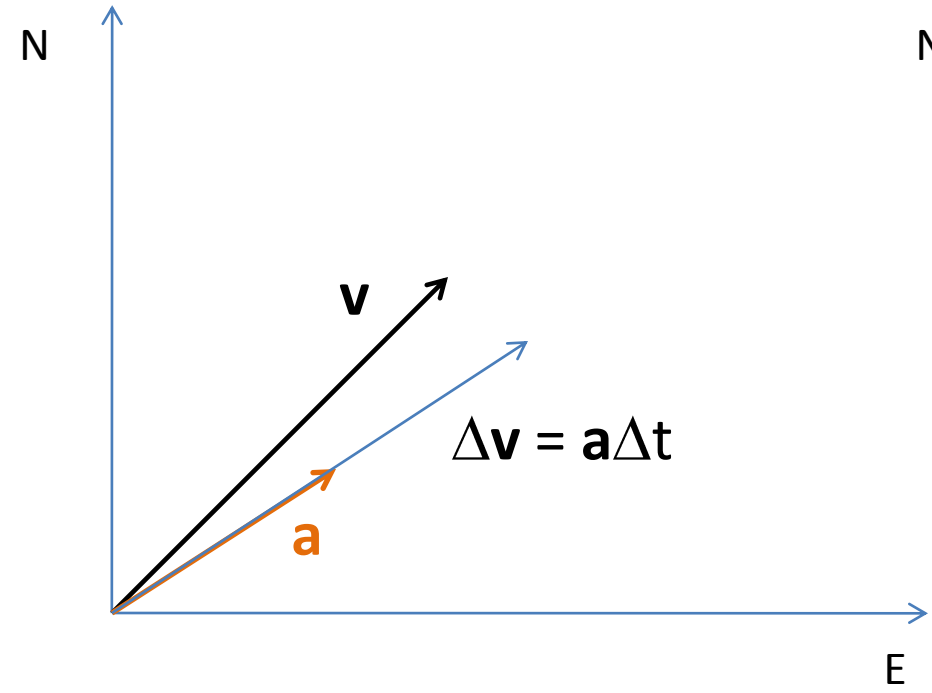
Pretend that the wind the blew on the boat's sail lasts for 2 seconds. We can now find the new velocity of the boat. Using $\Delta\mathbf{v} = \mathbf{a} \Delta t$.

$$\Delta\mathbf{v}_x = \mathbf{a}_x \Delta t: \quad (1.7 \text{ m/s}^2)(2 \text{ s}) = 3.4 \text{ m/s}$$

$$\Delta\mathbf{v}_y = \mathbf{a}_y \Delta t: \quad (1 \text{ m/s}^2)(2 \text{ s}) = 2 \text{ m/s}$$

$$\mathbf{v}_{fx} = \mathbf{v}_{ix} + \Delta\mathbf{v}_x = (2.8 + 3.4) \text{ m/s} = 6.2 \text{ m/s}$$

$$\mathbf{v}_{fy} = \mathbf{v}_{iy} + \Delta\mathbf{v}_y = (2.8 + 2) \text{ m/s} = 4.8 \text{ m/s}$$



Projectile motion

Objects that move in both the x and y directions.

Common examples:

baseballs, bullets, cannonballs, water fountains,
animals jumping, the General Lee jumping a creek...

(The list can go and on)

See fig 3.14 on page 62

Projectile Motion

When studying projectile motion, we will have gravity providing the acceleration. **With gravity pointing downward (-y direction), it has NO effect on the horizontal motion.**

$$a = g = -9.8 \text{ m/s}^2 \text{ (downward)}$$

This independence of the horizontal and vertical motion is very important.

One result is that, absent of outside forces the horizontal component of velocity will stay constant for projectiles.

Again refer to figure 3.14. Notice that the x-component of the velocity is constant while the y-component changes.

Horizontal Gunshot

If you shoot a gun horizontally. The bullet will leave the gun at some velocity. The horizontal component will stay constant, (the initial velocity).

The vertical component of velocity will begin to change (increase downwards) due to gravity.



This is why the bullet hit the target a bit lower than where the gun was aimed.

We will observe objects propelled sideways, up, at angles, and just dropped. What is important is that no matter the initial conditions (the initial velocity) the laws and formulas we will use apply to all.

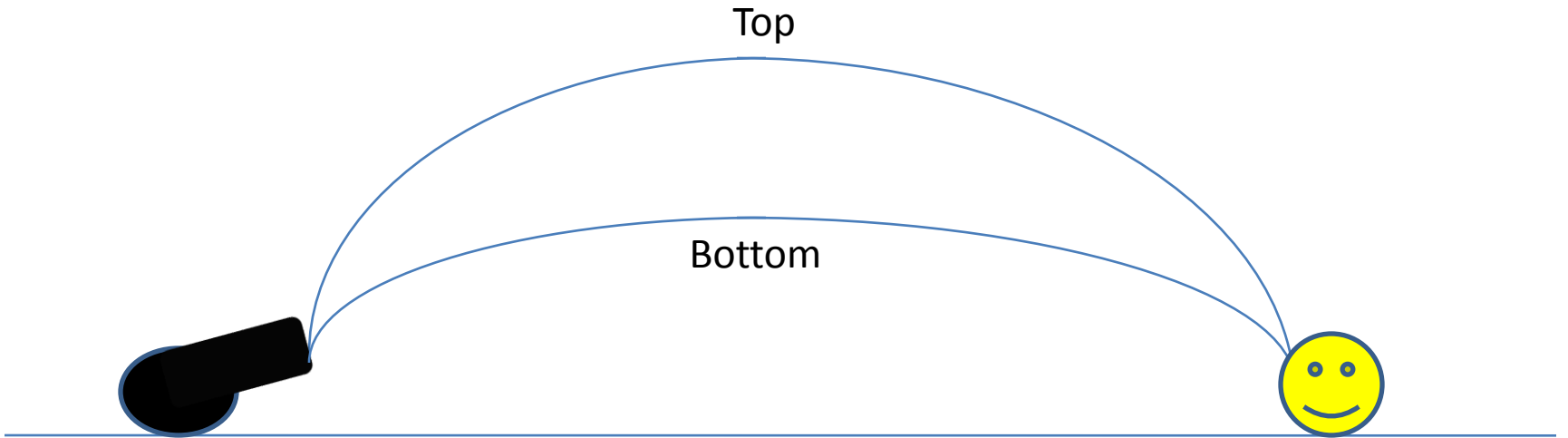
Projectile motion simulator

- <http://www.walter-fendt.de/ph11e/projectile.htm>

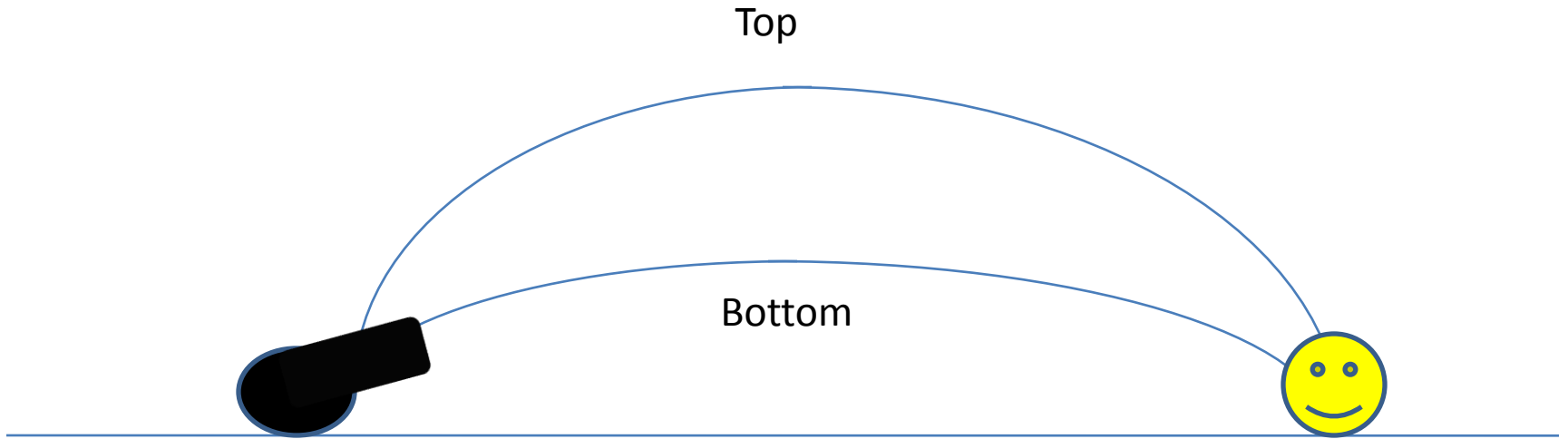
I'd like you to play around with this to see the effects of changing some of the variables.

Two Cannons

Which cannonball hits the target first?



Two Cannons



The cannonball that follows the bottom path hits first. It will need a higher x-component of velocity to hit the target.