

Work-Energy Part 2

Springs

Spring can be used to apply forces

Springs can store energy

These can be done by either compression,
stretching, or torsion.

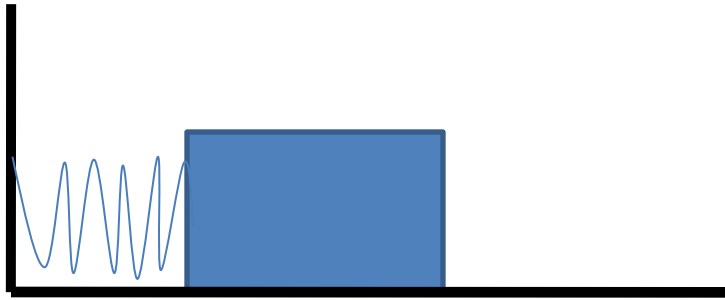
Springs

Ideal, or linear springs follow a rule called:

Hooke's Law: $F_s = -k x$

- k is called the spring constant. This determines how stiff the spring is.
- x is the distance the spring is deformed (stretched or compressed) from the equilibrium length.
- The minus sign tells us that this is a restoring force.

Restoring force means that the force the spring exerts, is in the opposite direction of the force that deforms the spring.



If I pull the box to the right (stretch the spring), the spring will exert a force to the left.

If I push the box to the left (compress the spring), the spring will exert a force to the right.

Spring constant

The spring constant, (k), determines how stiff a spring is.

- High spring constant

Stiff or strong, Hard to stretch or compress

- Low spring constant

Limp or weak, Easy to stretch or compress

- Units for spring constant are force per length:
N/m

Simple spring example

A spring with a spring constant of 250 N/m has a length of 0.5 meters when un stretched.

What magnitude of force is needed to stretch the spring so that is 0.75 meters long?

$$F_s = - k x$$

$$F_s = (250 \text{ N/m})(0.75\text{m} - 0.5\text{m}) = 62.5 \text{ N}$$

Note that it would take the same force to compress the spring by 0.25 meters.

Spring Work

$$W = F\Delta x$$

Because the spring force varies as we stretch/compress the spring, we have to use the average force between the two endpoints.

$$F_{\text{ave}} = (F_0 + F_1)/2 = (0 - kx)/2 = -\frac{1}{2} kx$$

Remember x is the change in the spring's length.

$$W_s = -\frac{1}{2} kx^2$$

See work done by varying forces on page 148.

Elastic Potential Energy

Energy you store up by changing a spring's length is: $PE_s = \frac{1}{2} kx^2$

This is the energy you store when doing work against a spring.

Conservation of Energy/Work Energy

What do you do when there are nonconservative forces, such as friction?

$$KE_i + PE_i = KE_f + PE_f$$

Becomes

$$KE_i + PE_i = KE_f + PE_f + \text{work done by } F_{nc}$$

Or

$$KE_i + PE_i = KE_f + PE_f + W_{nc}$$

$$KE_i + PE_i = KE_f + PE_f + W_{nc}$$

On the left hand side you have the initial mechanical energy.

On the right hand side you have the final mechanical energy PLUS the *magnitude* of the 'wasted' energy produced by nonconservative work.

Slide problem.

A 25 kg child climbs a 3 meter high ladder to get onto a slide. The slide has no friction. The length of the slide is 5 meters. Find the speed that the child reaches the ground with.

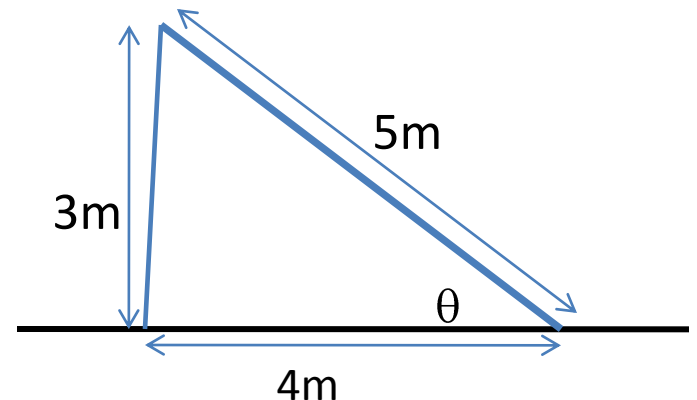
Use CoE (picked ground for zero PE line)

$$KE_i + PE_i = KE_f + PE_f$$

$$0 + mg\Delta y = \frac{1}{2}mv^2 + 0$$

$$(25\text{kg})g(3\text{m}) = \frac{1}{2}(25\text{kg})v^2$$

$$v = 7.7 \text{ m/s}$$



Slide problem with friction

Repeat previous problem with friction ($\mu_k = 0.5$)

Now use: $KE_i + PE_i = KE_f + PE_f + W_{nc}$

Find frictional force: $\mu F_n = 0.5(25\text{kg})g \cos \theta$

$$\cos \theta = 4/5$$

$$F_f = 98 \text{ N}$$

Magnitude of the Work done by friction is:

$$F_f \Delta x = (98 \text{ N})(5\text{m})$$

$$W_f = 490 \text{ J}$$

Now find the final speed of the child at the end of the slide.

$$KE_i + PE_i = KE_f + PE_f + W_{nc}$$

$$0 + (25\text{kg})g(3\text{m}) = \frac{1}{2} (25\text{kg})v^2 + 0 + 490 \text{ J}$$

$$735 \text{ J} = \frac{1}{2} (25\text{kg})v^2 + 490 \text{ J}$$

$$735 \text{ J} - 490 \text{ J} = 245 \text{ J} = \frac{1}{2} (25\text{kg})v^2$$

$$245 \text{ J} = \frac{1}{2} (25\text{kg})v^2$$

$$v = 4.4 \text{ m/s}$$

Recall that when there was no friction the final speed was 7.7 m/s.

More slide problem

Now repeat the problem with a longer slide. The height stays the same. But the slide is now 6 meters long. (This will change the angle to 30 degrees.)

Find magnitude of work done by friction.

$$F_f = \mu F_n = 0.5(25\text{kg})g (\cos 30) = 106 \text{ N}$$

$$\text{Magnitude of frictional work} = F_f \Delta x$$

$$W_f = (106 \text{ N}) 6\text{m} = 636 \text{ J}$$

$$KE_i + PE_i = KE_f + PE_f + W_{nc}$$

$$0 + (25\text{kg})g(3\text{m}) = \frac{1}{2} (25\text{kg})v^2 + 0 + 636 \text{ J}$$

$$735 \text{ J} = \frac{1}{2} (25\text{kg})v^2 + 636\text{J}$$

$$735 \text{ J} - 636 \text{ J} = 99 \text{ J} = \frac{1}{2} (25\text{kg})v^2$$

$$99 \text{ J} = \frac{1}{2} (25\text{kg})v^2$$

$$v = 2.8 \text{ m/s}$$

More slide problem

Now repeat the problem with an even longer slide. The height stays the same. But the slide is now 10 meters long. (This will change the angle to 17 degrees.)

Find magnitude of work done by friction.

$$F_f = \mu F_n = 0.5(25\text{kg})g (\cos 17) = 117 \text{ N}$$

$$\text{Magnitude of frictional work} = F_f \Delta x$$

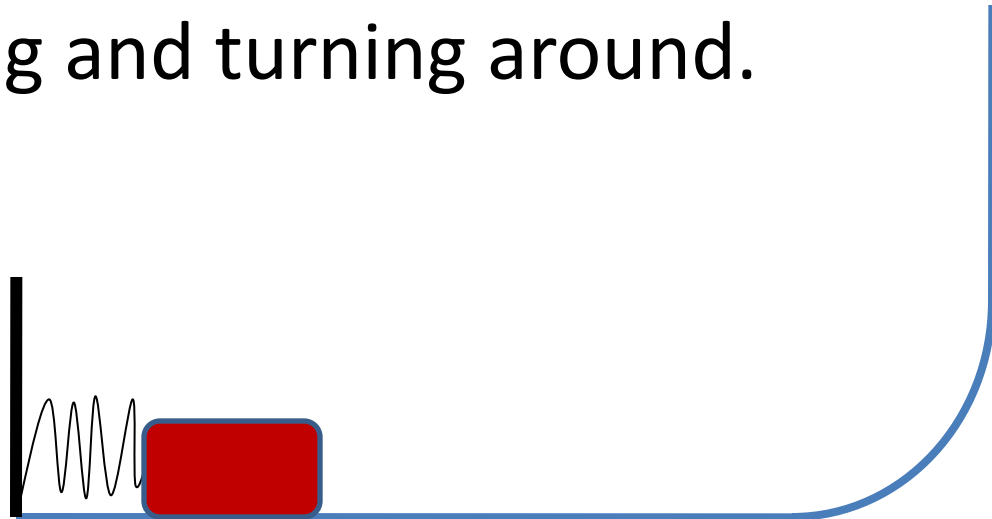
$$W_f = (117 \text{ N}) 10\text{m} = 1170 \text{ J}$$

Now the work done by friction is more than the initial potential energy. In this case, the child will not slide all the way down the slide.

The 735 J of potential energy is not enough to overcome all the friction needed to reach the end of the slide.

Cons. of energy problem.

A 2 kg block is compressing a horizontal spring with force constant ($k = 25 \text{ N/m}$) by a distance of 2 meters. When the spring releases the block, it follows a path as shown in the picture. Find out high the block will reach before stopping and turning around.



Use Cons. of Energy

I picked the ground level to be the zero of potential energy line.

Originally all the energy is spring potential:

$$PE_s = \frac{1}{2} kx^2$$

After the block leaves the spring, it will have kinetic energy:

$$KE = \frac{1}{2} mv^2$$

When block reached final height, it will have potential energy:

$$mg\Delta y$$

You want to find Δy

Initial energy is from the spring:

$$PE_s = \frac{1}{2} kx^2 = \frac{1}{2} (25 \text{ N/m})(2\text{m})^2 = 50 \text{ J}$$

When the block moves on the horizontal segment of the path it has kinetic energy:

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} (2\text{kg})v^2 = 50 \text{ J}$$

$$v = 7.1 \text{ m/s}$$

At the block's highest point, it has only gravity potential energy:

$$mg\Delta y = 50 \text{ J}$$

$$50 \text{ J} = (2 \text{ kg}) g \Delta y$$

$$\Delta y = 2.6 \text{ meters}$$

You could have skipped the middle step where we found how fast the block leaves the spring.

$$PE_s = PE_g$$

$$\frac{1}{2} kx^2 = mg\Delta y$$

$$\frac{1}{2} (25 \text{ N/m}) (2\text{m})^2 = (2 \text{ kg})g \Delta y$$

$$\Delta y = 2.6 \text{ meters}$$

Power

Definition of power is the work done per time.

$$P = W/\Delta t$$

SI unit of power is the watt. [J/s]

U.S. unit of power is horsepower

$$1 \text{ hp} = 746 \text{ W}$$

For electric power generation/usage, use the kilowatt-hour. This is the energy transferred in 1hr rate the rate of 1kW (1000 J/s).

$$1\text{kWh} = (1000\text{J/s})(3600\text{s}) = 3.6 \times 10^6 \text{J}$$

Remember Work = $F\Delta x$

$$P = F\Delta x/\Delta t \quad \text{so} \quad P = F v \quad v = \text{velocity}$$

Power is a scalar quantity.

$$P_{\text{ave}} = F v_{\text{ave}}$$

Average power is what is needed to apply a force on an object to give that object a corresponding average velocity.

Power

Power is related to how fast a force can be applied.

Picking up a heavy weight slowly may not require much power.

Picking up the same weight quickly will require more power.

Weightlifting examples: compare the power required to perform a bench press and an Olympic snatch.

Assume the weights are moved at constant velocities.

Bench press: 300 lbs (1335 N),

Range of motion $\Delta x \sim 0.5 \text{ m}$

time to raise weight $\sim 3 \text{ seconds}$

$$P = F\Delta x/\Delta t \quad (1335 \text{ N})(0.5 \text{ m})/(3\text{s}) = 222.5 \text{ W}$$

Olympic snatch: 100 lbs (445 N)

Range of motion $\Delta x \sim 1.5 \text{ m}$

time to raise weight $\sim 1 \text{ s}$

$$P = F\Delta x/\Delta t \quad (445 \text{ N})(1.5 \text{ m})/(1\text{s}) = 667.5 \text{ W}$$

Even though the snatch is performed with less weight, it requires more power because of the larger velocity.

Bucket Example

You want to lift a 20 kg bucket up a well at a constant velocity of 0.5 m/s. What power is needed to do so?

Since the velocity is constant, the upward force you must pull with is equal to the weight of the bucket.

$$P = F v = (20\text{kg})g (0.5 \text{ m/s}) = 98 \text{ W}$$

Another bucket example

Again you want to raise the same 20 kg bucket. It is starting from rest, and you want to pull on the bucket so that it has a velocity of 2 m/s. You want to accomplish this over a time interval of 4 seconds.

Use Power and the work energy theorem.

(net work equals change in kinetic energy.)

first find the change in the kinetic energy.

$$\Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = \frac{1}{2} (20\text{kg})(2\text{m/s})^2$$

$$\Delta K = 40 \text{ J}$$

$$\text{Work} = 40 \text{ J}$$

$$\text{Power} = \text{Work}/\text{time} = (40 \text{ J}) / (4\text{s}) = 10 \text{ W}$$

Shamu example

Calculate the average power needed for the whale to speed up. The killer whale has mass of 8000 kg

What power is needed to reach speed of 12m/s in a 6 second time interval?

Do work energy theorem

$$\Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = \frac{1}{2} (8000\text{kg})(12\text{m/s})^2$$

$$\Delta K = 5.76 \times 10^5 \text{J}$$

$$\text{Power} = (5.76 \times 10^5 \text{J}) / 6 \text{ s} = 9.6 \times 10^4 \text{ W}$$

This was neglecting drag. The power is actually higher.

$$9.6 \times 10^4 \text{ W} (1\text{hp}/746\text{W}) = 128 \text{ hp}$$

About the same as a car.

Power delivered by elevator motor

A 1000 kg elevator carries a load of 800 kg. A constant friction force of 4000 N retards its upward motion. The retarding force behaves similarly to friction. What minimum power in kilowatts and horsepower must the motor deliver to lift the fully loaded elevator at a constant speed of 3 m/s?

Elevator

Since the speed is constant (3m/s), the acceleration and the sum of all forces equals zero.

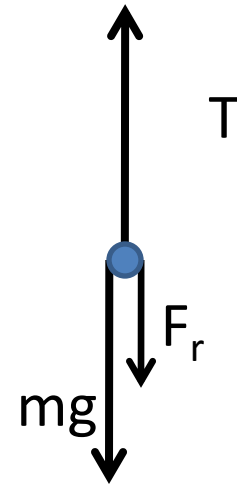
Find the force (T) the motor must pull with to achieve this motion.

$$\Sigma F = ma = 0$$

$$T - F_r - mg = 0$$

$$T = F_r + mg = 4000 \text{ N} + (1800\text{kg})g$$

$$T = 2.16 \times 10^4 \text{ N}$$



Elevator

Now that we know the force the motor must exert we can find the power using $P = Fv$.

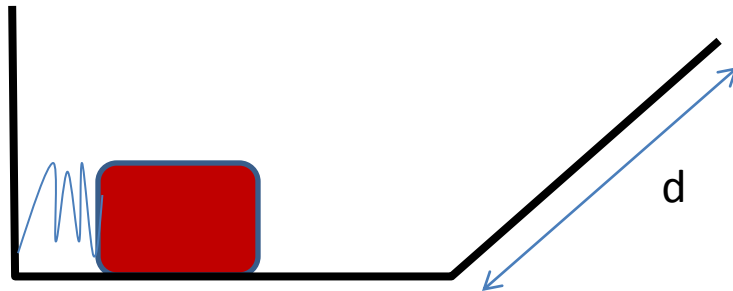
$$P = Fv = (2.16 \times 10^4 \text{ N})(3 \text{ m/s}) = \underline{6.48 \times 10^4 \text{ W}}$$

$$746 \text{ W} = 1 \text{ hp}$$

$$\text{So } P = (6.48 \times 10^4 \text{ W}) \times (1 \text{ hp}) / (746 \text{ W}) = \underline{87 \text{ hp}}$$

Using a spring to propel a block up a ramp.

A spring with force constant 500 N/m is compressed 0.8 meters, with a block what has a mass of 45 kg . After the spring 'shoots' the block it travels up an 45 degree incline. How far along the incline will the block travel?



Do conservation of energy

Initially all the energy is spring potential energy.

$$PE_s = \frac{1}{2} kx^2 = \frac{1}{2} (500 \text{ N/m})(0.8\text{m})^2 = 160 \text{ J}$$

After going up the ramp, when the block stops, all the energy is from gravitational potential energy.

$$PE_g = mgh \quad 160 \text{ J} = (45\text{kg})g h$$

$$h = 0.36 \text{ m}$$

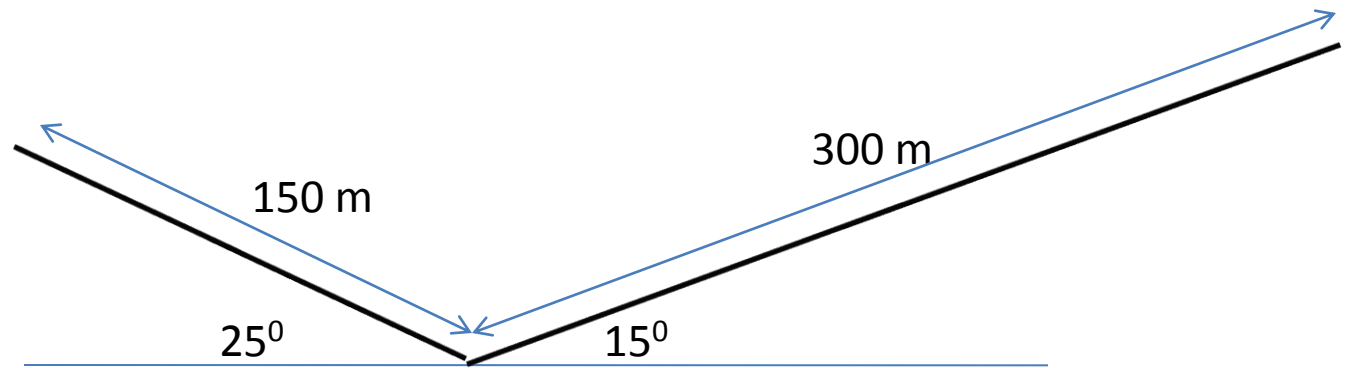
Want distance along the incline (d).

$$d \sin \theta = h \quad d \sin 45 = 0.36 \text{ m}$$

$$\underline{d = 0.51 \text{ m}}$$

Runaway truck

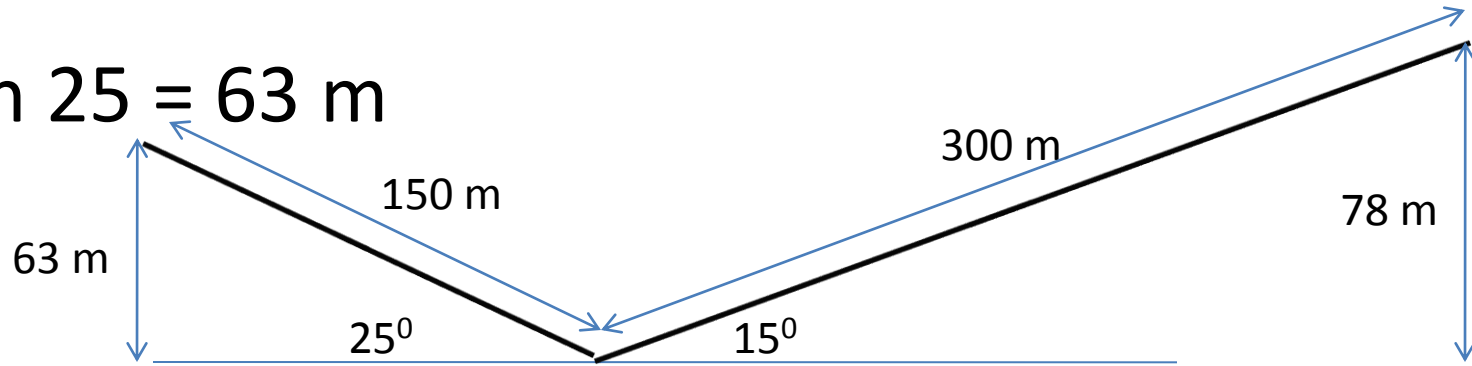
A runaway truck is driving 25 m/s down a hill. The truck has 300 meters to go before reaching the bottom and the hill is on a 15 degree angle. At the bottom of the hill is a runaway truck ramp. The ramp is 150 meters long at a 25 degree angle. Will the ramp stop the truck?



First find the important heights.

$$300 \text{ m} \sin 15 = 78 \text{ m}$$

$$150 \text{ m} \sin 25 = 63 \text{ m}$$



Do conservation of energy.

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2} m v_i^2 + mgh_i = \frac{1}{2} m v_f^2 + mgh_f$$

$$\frac{1}{2} m (25\text{m/s})^2 + mg(78\text{m}) = mg(63\text{m}) + \frac{1}{2} m v_f^2$$

$$\frac{1}{2} (25\text{m/s})^2 + g(78\text{m}) = g(63\text{m}) + \frac{1}{2} v_f^2$$

$v_f = 30.3 \text{ m/s}$ The truck is moving faster.

Runaway truck ramps will have a rough surface (usually in addition they have piles of sand on them) to produce a lot of friction. If we include a coefficient of friction of 0.4 between the truck and the ramp, will the ramp stop the truck?

Runaway truck with friction

Use work energy/conservation of energy

$KE_i + PE_i = KE_f + PE_f + W_{nc}$, where W_{nc} is the work done by friction.

$$\frac{1}{2} m v_i^2 + mgh_i = \frac{1}{2} m v_f^2 + mgh_f + \mu_k mg(\cos \theta)d$$

$$\frac{1}{2} m(25\text{m/s})^2 + mg(78\text{m}) = mg(63\text{m}) + \frac{1}{2} m v_f^2 + 0.4mg(\cos 25)(150\text{m})$$

$$\frac{1}{2}(25\text{m/s})^2 + g(78\text{m}) = g(63\text{m}) + \frac{1}{2}v_f^2 + 0.4g(\cos 25)(150\text{m})$$

$$313 + 764 = 617 + 533 + v_f^2$$

$$1077 = 1150 + \frac{1}{2} v_f^2$$

v_f^2 can't be negative so the truck was stopped.

We can find how far along the ramp,
the truck stops.

$$KE_i + PE_i = KE_f + PE_f + W_{nc},$$

$$\frac{1}{2} m v_i^2 + mgh_i = \frac{1}{2} m v_f^2 + mgh_f + \mu_k mg(\cos \theta)d$$

Change h_f to $d \sin 25$ since we can relate the final height to the distance along the ramp

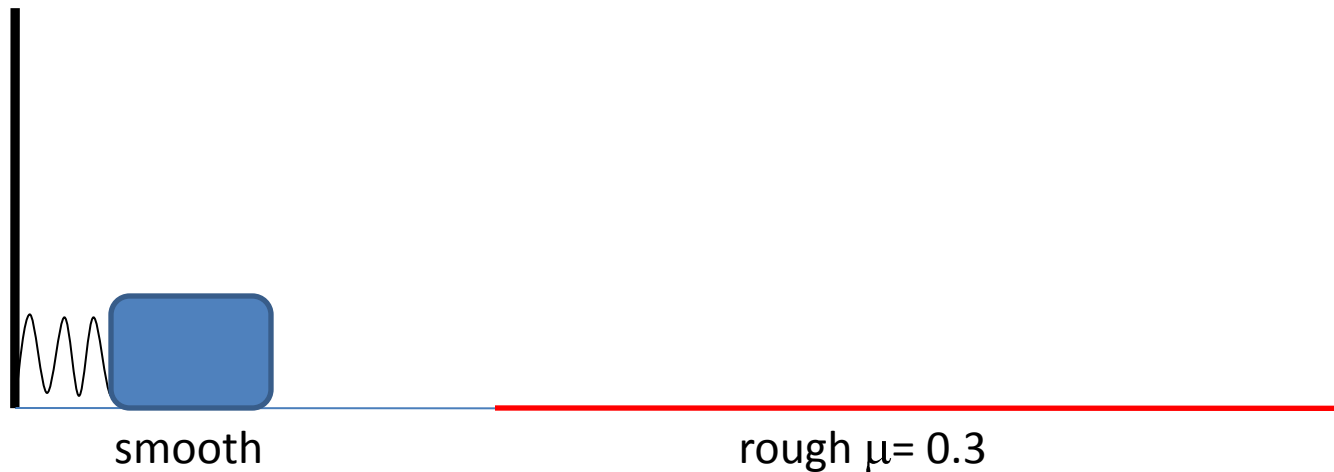
$$\frac{1}{2} (25\text{m/s})^2 + g(78\text{m}) = 0 + g d \sin 25 + 0.4g(\cos 25)d$$

$$1077 = d (g \sin 25 + 0.4 g \cos 25)$$

$$1077 = d (7.7)$$

$$d = 140 \text{ m}$$

A 50 kg block compresses a spring ($k = 1000\text{N/m}$) a distance of 0.5 meters. After the block is released, it moves on a frictionless surface for 2 meters before sliding on a rough surface. If the coefficient of friction is 0.3, how far along on the rough surface, does the block slide?



- Initial situation:

Spring potential energy $PE_s = \frac{1}{2} kx^2$

The block eventually comes to rest. The rough surface does work equal to the initial energy.

remember: $W_f = F_f d = \mu mgd$

$$\frac{1}{2} kx^2 = \mu mgd$$

$$\frac{1}{2} (1000 \text{ N/m})(0.5 \text{ m})^2 = 0.3 (50 \text{ kg}) g d$$

$$d = 0.85 \text{ m}$$