Chapter 6
Momentum and Collisions

Conceptual questions: 3, 4, 6, 11, 14

Problems: 1, 2, 3, 4, 7, 9, 11, 19, 20, 22, 24, 37, 39, **40**, 57, 59

Momentum

Momentum is the product of object's mass and its velocity.

$\mathbf{p} = m \mathbf{v}$

Momentum is a vector quantity
(direction is important).

Momentum is in the same direction as the object's velocity.

The units of momentum are: kg m/s

Examples

A 1000 kg truck moving with velocity 30 m/s

p = (1000 kg)(30 m/s) = 3×10^4 kg m/s

A 3000 kg truck moving with velocity 30 m/s

p = (3000 kg)(30 m/s) = 9×10^4 kg m/s

A 0.14 kg baseball with velocity 40 m/s

p = (0.14 kg)(40 m/s) = 5.6 kg m/s

A 6 kg bowling ball with velocity 5 m/s

p = (6 kg)(4 m/s) = 24 kg m/s

Momentum and Impulse

$\mathbf{F}_{net} = m\mathbf{a}$ and: $\mathbf{a} = \Delta v/\Delta t$

$F_{net} = m\,\Delta v/\Delta t$

Use momentum $\mathbf{p} = m\mathbf{v}$

So $\mathbf{F}_{net} = \Delta\mathbf{p}/\Delta t$

Impulse $\mathbf{I} = \mathbf{F}\Delta t$

So $\mathbf{I} = \Delta\mathbf{p}$

Impulse is equal to the change in momentum.

Impulse-momentum theorem

$\mathbf{F}\Delta t = \Delta\mathbf{p} = m\mathbf{v}_f - m\mathbf{v}_i$

The impulse of the force acting on an object equals the change in momentum of that object.

Work example 6.2

Impulse from non-constant force

The magnitude of the impulse delivered by a force over the time interval Δt, is equal to the area under the force vs. time graph as in Figure 6.1a.

This is equivalent to the area $\mathbf{F}_{ave}\Delta t$ as shown in figure 6.1b.

$$\mathbf{F}_{ave}\Delta t = \Delta \mathbf{p}$$

See page 163.

- A 0.3 kg soccer ball sits at rest before being kicked. The ball is in contact with the foot for 0.3 seconds and has a final with a velocity of 10 m/s. What is the average force exerted on the ball?

$\mathbf{F}_{ave}\Delta t = \Delta \mathbf{p}$

$\mathbf{F}_{ave} = (\mathbf{p}_f - \mathbf{p}_i)/\Delta t = ((0.3\text{kg})(10\text{m/s}) - 0)/(0.3\text{ s})$

$\mathbf{F}_{ave} = 10\text{ N}$

Conservation of momentum

When a collision occurs in an isolated system, the individual momentums of objects my change, but the total momentum vector of the whole system is a constant.

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

There could be more than two particles. If that is the case, we just include as many initial and final momentum terms as we need.

Conservation of momentum

When there is no net total force acting on a system, the total momentum of the system remains constant with time.

Archer example

An archer stands on frictionless ice and shoots a 0.5kg arrow horizontally at 50 m/s. The combined mass of the archer and the bow is 60 kg. With what velocity will the archer move after shooting the arrow?

Use conservation of momentum

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

Archer

Initially the total momentum is zero. The initial velocities of the archer and the arrow are both zero.

So after the arrow is shot, the total momentum must still be zero.

Archer

(labeled the archer as 1 and the arrow as 2)

$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$

$(60kg)0m/s + (0.5kg)0m/s = (60kg)v_{1f} + (0.5kg)(50m/s)$

$0 = (60kg)v_{1f} + (0.5kg)(50m/s)$

$(60kg)v_{1f} = -(0.5kg)(50m/s)$

$v_{1f} = -0.417\ m/s$

If the arrow is shot to the right, the archer moves to the left.

Collisions

- When objects collide, momentum is always conserved.

- Kinetic energy is not always conserved. Some of the kinetic energy is turned into sound, work needed to deform the object, heat, internal energy.

Collisions

- We will be looking at two types of collisions.

- Perfectly elastic collisions
  - momentum and kinetic energy is conserved
  - the objects leave each other with the same speed as they hit each other
  - there is no deformation of the objects during the collision
  - examples:
    - Pool balls (approximately)
    - Proton-proton collision

Collisions

- Perfectly Inelastic collisions
  - momentum is conserved, but kinetic energy is lost
  - the objects stick together after the collision. (they move with the same velocity)
  - examples:
    - Two cars hitting each other and the bumpers lock up
    - Football player being tackled by another player
    - Person catching a ball
    - Shooting a ball into a ballistic pendulum

Collisions

Nearly all collisions in the real world fall somewhere in between being perfectly elastic and perfectly inelastic.

When you hit a baseball the ball doesn't stick to the bat, so the collision isn't perfectly inelastic.

Some of the kinetic energy is converted to sound and is used to momentarily deform the ball and bat, so the collision isn't perfectly elastic either.

Perfectly Inelastic Collisions

Since the two objects have the same velocity after the collision, we can rewrite the conservation of energy as:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$$

Solving for the final velocity gives us:

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Remember to treat v_{1i} and v_{2i} as vectors. Their direction matters.

Inelastic Collision.

Shoot a 0.05 kg bullet at 300 m/s into a 5kg wooden block. The bullet gets stuck in the block, so they move together afterwards. Find the final velocity of the bullet/block.

Cons. of Mom. says: $m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$

Let the bullet be mass 1 and the block be mass2.

$$v_f = \frac{(0.05kg)(300\,m/s) + (5kg)(0m/s)}{0.05kg + 5kg} = 2.97\,m/s$$

Perfectly Elastic Collision

Momentum is conserved

$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$

Kinetic Energy is conserved

$\frac{1}{2} m_1\mathbf{v}_{1i}^2 + \frac{1}{2} m_2\mathbf{v}_{2i}^2 = \frac{1}{2} m_1\mathbf{v}_{1f}^2 + \frac{1}{2} m_2\mathbf{v}_{2f}^2$

Combining these two equations, if I gave you the initial velocities of the objects you could solve for the two final velocities simultaneously.

Elastic Collisions

Do a bunch of algebra and you will get:

$$v_{1f} = \frac{(m_1 - m_2)v_{1i} + 2m_2 v_{2i}}{m_1 + m_2}$$

and

$$v_{2f} = \frac{2m_1 v_{1i} + (m_2 - m_1)v_{2i}}{m_1 + m_2}$$

Some visual examples of elastic collisions

- http://en.wikipedia.org/wiki/Elastic_collision

Glancing Collisions

Remember that momentum is a vector. We can expand our work into 2 dimensions.

All you need to realize is that the momentum in the x-direction is conserved.

In addition, the momentum in the y-direction is conserved.

http://www.youtube.com/watch?v=KteP6k1wg94

Rocket Propulsion

Rockets work using the law of conservation of momentum.

Remember how the archer could propel himself across the lake by shooting an arrow?

Rockets work in a similar manner. When expelling gas in one direction, in order for momentum to be conserved, the rocket must move in the opposite direction.

Rockets

A rocket starts with an initial mass. This includes the fuel that will be used. As the rocket burns the fuel, the total mass of the rocket is reduced. Therefore the mass of the rocket that is being propelled changes with time. If we want to show how to calculate the final speed of the rocket after a known amount of fuel is used up, we would use calculus.

Rocket

Skipping the derivation we get the equation:

v_e is the exhaust velocity of the gas relative to the rocket

M_i and M_f are the initial and final masses of the rocket.

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$$

Ballistic Pendulum

A 0.05 bullet with velocity 150 m/s is shot into a 3 kg ballistic pendulum. Find how high the pendulum rises after the bullet gets stuck inside.

Ballistic Pendulum

First use conservation of momentum to find the final velocity of the bullet/pendulum.

This is an inelastic collision.

$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$

$(0.05\text{kg})(150\text{m/s})+(3\text{kg})(0\text{m/s}) = (3.05\text{kg})v_f$

$v_f = 2.5$ m/s

Ballistic Pendulum

Now that we know the ballistic pendulum with the bullet in it begins to swing with a speed of 2.5 m/s, we use conservation of energy to find how high it swings.

$\frac{1}{2} mv^2 = mg\Delta y$

$\frac{1}{2} m (2.5 m/s)^2 = mg\Delta y$

$\Delta y = 0.32$ m

Exploding Bird

A 2 kg bird is flying in the positive x-direction with a speed of 5 m/s. The bird explodes into two pieces. A 1.5 kg piece of bird has velocity of 4 m/s in the positive y-direction. What is the velocity of the 0.5 kg piece?

Use conservation of momentum

Exploding Bird

Need to do conservation of momentum in both the horizontal and vertical directions.

Initially the horizontal momentum is:

(2kg)(5m/s) = 10kg m/s

Initial vertical momentum is zero.

So the final momentum will be 10 kg m/s in the horizontal direction.

Initial momentum: 10 kg m/s x-direction

Momentum of 1.5 kg piece:

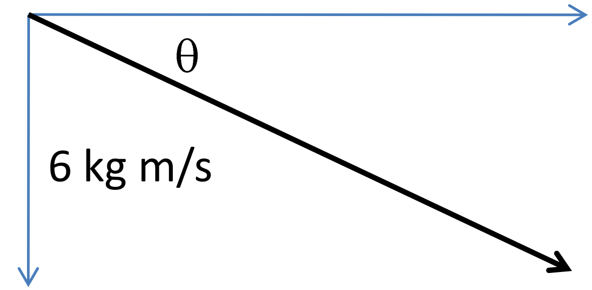
(1.5kg)(4m/s) = 6 kg m/s in positive y-direction

For momentum to be conserved, the 0.5 kg piece must have momentum in both the x and y directions.

Since the 1.5 kg piece had no x-momentum, the 0.5 kg piece must have 10 kg m/s of momentum in the x-direction.

The vertical momentum of the two pieces must cancel out, so the y-momentum of the 0.5 kg piece must be 6 kg m/s in the negative y-direction.

Momentum of 0.5 kg piece:



Total momentum of 0.5 kg piece is:

$$\sqrt{(6kgm/s)^2 + (10kgm/s)^2} = 11.7kgm/s$$

Get the angle θ: $\theta = \tan^{-1}(6/10) = 31$ degrees

Now divide the momentum by the mass to get the velocity.

p = m**v** **v** = **p**/m = (11.7kg m/s)/(0.5kg)

v has magnitude of 23.4 m/s. The angle is $\theta=31^0$