

# Chapter 7

Concept questions# 5, 10

Problems# 3, 5, 7, 9, 15, 16, 21, 27,  
32, 33, 43, 66, 76

$\theta = s / r$ , where  $s$  is the arc length and  $r$  is the radius.

$\theta$  is measured in radians or degrees.

360 degrees is  $2\pi$  radians.

1 revolution =  $2\pi$  radians

1 radian is about 57.3 degrees.

See figures on page 191.

Do quick quizzes 7.1, 7.2

# Rotational motion

Earlier we did linear motion:

position  $x$

linear displacement  $\Delta x$

linear velocity  $v$

linear acceleration  $a$

Now we look at rotational motion

angular position  $\theta$

angular displacement  $\Delta\theta$

angular velocity  $\omega$

angular acceleration  $\alpha$

These behave exactly like their linear counterparts.

$$v = v_0 + at$$

$$\Delta x = v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

Become:

$$\omega = \omega_0 + \alpha t$$

$$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

# Relations between angular and linear quantities.

$$\Delta\theta = \Delta s/r$$

$$\omega = v/r$$

Tangential speed  $v_t$

$$v_t = r\omega \quad \text{or } \omega = v_t/r$$

Tangential acceleration

$$a_t = r\alpha \quad \text{or } \alpha = a_t/r$$

do quick quizzes 7.4, 7.5

Work example 7.4

# Centripetal Acceleration

- $a_c = v^2/r$
- This is the acceleration you get when traveling in a circle.
- Substituting  $\omega = v/r$
- $a_c = \omega^2 r$
- Total acceleration includes the tangential and the centripetal accelerations.

$$a = \sqrt{a_t^2 + a_c^2}$$

quick quizzes 7.6, 7.7

# Angular quantities $\omega$ and $\alpha$ are vectors.

- Use Right Hand Rule.
  - Curl your fingers in direction of motion. Thumb will point in direction of  $\omega$ .
- $\omega$  and  $\alpha$  vectors are actually perpendicular to the motion.

A force must act on an object to produce a centripetal acceleration.

For example: a string tied to a swinging ball.

Centripetal force is not a real force.

$$F_c = ma_c = mv^2/r$$

What you really did was do Newton's 2<sup>nd</sup> law.

And sub in  $v^2/r$  for the  $a$  in  $F = ma$ .

work examples 7.7 and 7.8 and exercise 7.9 on page 207

# Gravitation.

- Earlier we saw how to measure the force of gravity between 2 objects.

$$F = G \frac{m_1 m_2}{r^2}$$

Now we can find the gravitational potential energy using this force.

PE = mgh becomes  $PE = -G \frac{M_E m}{r}$

- $PE = mgh$  when the change in heights are relatively small compared to Earth's radius.
- $PE = -G \frac{M_E m}{r}$  when the change in heights are not small compared to the Earth's radius.
- This is because the gravitational force varies when the altitude varies considerably.

# Escape velocity.

- You can figure out what velocity is needed to escape the gravitational pull of a planet.
- This is the initial velocity needed for an object to reach a distance of infinity away from the planet.
- Using conservation of energy:

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mv_{esc}^2 - \frac{GM_E m}{R_E} = 0$$

Escape velocity becomes:

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

# Kepler's Laws

Laws that determine orbital motion.

1<sup>st</sup> Law: Planets orbit in elliptical orbits with the sun being located at one of the focal points.

2<sup>nd</sup> Law: Lines drawn from the sun to the planet sweeps out equal areas over equal time intervals.

3<sup>rd</sup> Law: The square of the orbital period is proportional to the cube of the average distance from the planet to sun.

# Kepler's 3<sup>rd</sup> Law.

Equating the gravitational force to the centripetal force gives:

$$\frac{m_p v^2}{r} = G \frac{M_s m_p}{r^2}$$

Substituting  $v = 2\pi r/T$  and then doing some algebra we get:

$$T^2 = \left( \frac{4\pi^2}{GM_s} \right) r^3$$

Do example 7.14

Problem # 41

# Tarzan swings on a vine

- Tarzan  $m = 85 \text{ kg}$
- vine  $10 \text{ m}$
- Speed at bottom of swing is  $8.0 \text{ m/s}$
- Breaking strength is  $1000 \text{ N}$

Does vine break?

Use  $\Sigma F = ma$

$a$  gets replaced by  $v^2/r$

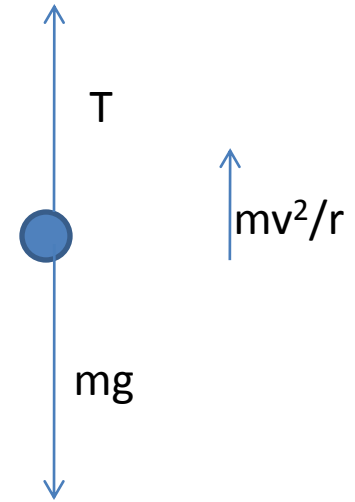
$$T - mg = mv^2/r$$

$$T = mg + mv^2/r$$

$$T = (85\text{kg})g + (85\text{kg}) (8\text{m/s})^2/(10\text{m})$$

$$T = 1377 \text{ N}$$

vine breaks



# Car Driving around curve

- Let friction coefficient between tires and road be  $\mu = 0.9$ . The curve of the road has a radius of 30 meters. How fast can the car take the curve without sliding off its circular path?

The friction is the force that needs to be 'balanced' out by the centripetal force.

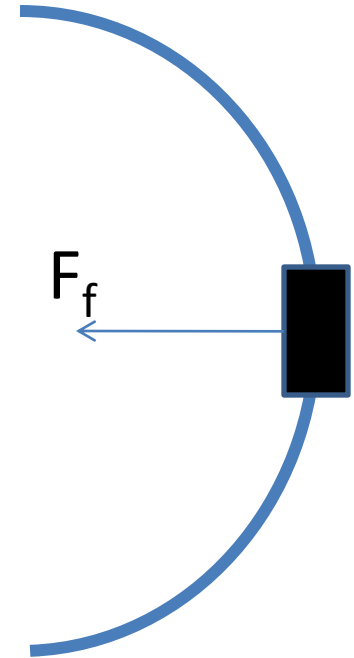
$$\Sigma F = mv^2/r$$

$$\mu mg = mv^2/r$$

$$0.9 mg = m v^2/(30 m)$$

$$v = \sqrt{0.9gr} = 16.3 \text{ m/s}$$

Looking down over the road.



Any faster and the car can't hold the turn.