

Ch.8 Rotational Equilibrium and Rotational Dynamics.

Conceptual question # 1, 2, 9

Problems# 1, 3, 7, 9, 11, 13, 17, 19, 25,
28, 33, 39, 43, 47, 51, 55, 61, 79 83

Torque

Force causes acceleration.

Torque causes angular acceleration.

Torque causes an object to change its rotational state.

$$\tau = rF \quad (\text{when } r \text{ and } F \text{ are perpendicular})$$

SI unit Newton-meter (N m)

When r and F are not perpendicular.

$$\tau = r F \sin \theta$$

θ is the angle between the r and F vectors

see examples on pg. 230

See example 8.1

The vector component of the force that is perpendicular to the moment arm (r) provides the torque.

The vector component of the force that is parallel to the moment arm produces no torque.

Right Hand Rule

Point fingers in direction of \mathbf{r} .

Curl fingers towards direction of \mathbf{F} .

Thumb points in direction of torque.

or

Index finger points in direction of \mathbf{r} .

Middle finger points in direction of \mathbf{F} .

Thumb points in direction of torque.

Counterclockwise torques are positive.

Torque and Conditions of Equilibrium.

Net external forces must be zero. $\Sigma F = 0$

No linear acceleration.

Net external torques must be zero. $\Sigma \tau = 0$

No angular acceleration.

see example 8.2

Center of Gravity

- If you want to take a rigid body and represent it as a point object. You would want to find which point to use. That is the center of mass. Or center of gravity.

$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i}$$

$$\bar{y} = \frac{\sum m_i y_i}{\sum m_i}$$

- Quite often, there are symmetry axes that make life easy. The center of mass will lie on a symmetry axis.

Weighted Forearm Example

Page 237.

We want to find the upward force exert by the biceps and the downward force exert by the humerus.

Forearm is in equilibrium.

Apply condition of net force equals zero.

Apply condition of net torque equals zero.

- $\Sigma F = 0$

$$F - R - W_{\text{ball}} = 0$$

$$F - R - 50 \text{ N} = 0$$

- $\Sigma \tau = 0$

$$\tau_R + \tau_F + \tau_{\text{ball}} = 0$$

Pick the elbow joint as the pivot point.

$$R(0) + F(0.03 \text{ m}) - (50\text{N})(0.35\text{m}) = 0$$

Two equations and two unknowns. Solve for R and F.

$$F - R - 50 \text{ N} = 0$$

$$R(0) + F(0.03 \text{ m}) - (50\text{N})(0.35\text{m}) = 0$$

$$R = 533 \text{ N}$$

$$F = 583 \text{ N}$$

Torque and Angular Acceleration.

- $F = ma$
- Now we have want to relate τ and α .

$$F = m a$$

$$F r = m r a$$

$$\text{Use: } a = r \alpha$$

$$F r = m r^2 \alpha$$

$$\tau = m r^2 \alpha$$

$m r^2$ is called the moment of inertia (I)

Moment of Inertia

For a point object:

The moment of inertia is: $I = mr^2$

m is the object mass and r is the distance to the axis of rotation.

For a rigid body made up of more than one point, we need to add up the moments of inertia for each point.

$$I = \sum m_i r_i^2$$

The angular acceleration of the rigid body is proportional to the net torque acting on it.

The proportionality factor is the moment of inertia.

$$\Sigma \tau = I \alpha$$

On page 244 we can see what the moment of inertia is for different shaped objects.

Work example 8.10, 8.11

Rotational Kinetic Energy

- $KE = \frac{1}{2} mv^2$ for objects with translational velocity.
- When objects rotate, they also have kinetic energy.

$$KE_r = \frac{1}{2} I\omega^2$$

So for a ball rolling down a hill, there are two components to the kinetic energy.

$$KE = KE_t + KE_r = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$$

Ball rolling down an incline.

- Solid ball of mass M and radius R starts from rest at a height of 2 m and rolls down 30 degree slope. What is the linear speed at the bottom.

Use Cons. of Energy

Old way: $K_i + P_i = K_f + P_f$

$$0 + Mgh = \frac{1}{2} Mv^2 + 0$$

$$v = \sqrt{2gh}$$

- Now we include the rolling of the ball. We need to include the rotational kinetic energy.

Cons. of Energy

$$K_{ti} + K_{ri} + P_i = K_{tf} + K_{rf} + P_f$$

$$0 + 0 + Mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 + 0$$

$$Mgh = \frac{1}{2} mv^2 + \frac{1}{2} (2/5 MR^2)\omega^2$$

$$Mgh = \frac{1}{2} mv^2 + 1/5 MR^2 (v/R)^2$$

$$Mgh = (7/10)Mv^2$$

$$v = \sqrt{\frac{10gh}{7}}$$

When we neglected the rotation of the ball we got a final velocity of $v = \sqrt{2gh}$

When we included the rotation of the ball we found a final velocity of $v = \sqrt{\frac{10gh}{7}}$

This tells us that different shaped objects will reach different velocities after rolling down a ramp.

Can repeat with a hollow ball, ring, disk.

Angular Momentum

Linear Momentum $\mathbf{p} = m\mathbf{v}$

Angular momentum $\mathbf{L} = I\boldsymbol{\omega}$

Angular momentum depends on the moment of inertia and the angular velocity.

Remember: $\mathbf{F} = \Delta\mathbf{p}/\Delta t$

Likewise: $\boldsymbol{\tau} = \Delta\mathbf{L}/\Delta t$

Angular Momentum is a Conserved quantity.

- Conservation of linear momentum
- $\mathbf{p}_i = \mathbf{p}_f$

- Conservation of angular momentum
- $\mathbf{L}_i = \mathbf{L}_f$
$$I_i \omega_i = I_f \omega_f \quad \text{if} \quad \Sigma \tau = 0$$

Do angular momentum demonstrations.

P. 8.85

Iron Cross: Gymnast weighing 750 N.

Each ring supports half the weight.

F_s is force exerted by the shoulder joint.

F_m is the force exerted by the pectoralis major and latissimus dorse.

F_m acts 4 cm from the shoulder at a downward 45 degree angle.

The arm is 70 cm long.

Estimate force F_m .

Apply the condition of $\Sigma\tau = 0$,
where the pivot point is the shoulder.

$$(w/2)(70 \text{ cm}) - (F_m \sin 45)(4 \text{ cm}) = 0$$

$$F_m = \frac{w(70 \text{ cm})}{2(4 \text{ cm}) \sin 45} = 12.4w = 9300 \text{ N}$$

P 8.52

Use conservation of energy to determine the angular speed of the spool, after the 3 kg bucket has fallen 4 m, starting from rest.

$$m_b = 3 \text{ kg}$$

$$m_s = 5 \text{ kg}$$

$$r_s = 0.6 \text{ m}$$

Using conservation of energy.

$$K_i + P_i = K_f + P_f$$

Pick zero of potential energy line to be where the bucket is lowered to.

Initial kinetic energy = 0

Final kinetic energy is from the motion of the bucket and the spinning of the wheel.

$$0 + m_b gh = \frac{1}{2} m_b v_b^2 + \frac{1}{2} I_s \omega^2$$

$$0 + m_b gh = \frac{1}{2} m_b v_b^2 + \frac{1}{2} I_s \omega^2$$

The velocity of the bucket is related to the angular velocity of the spool by $v = r\omega$.

treat spool as a disk $I_s = \frac{1}{2} m r^2$

$$0 + m_b gh = \frac{1}{2} m_b (r\omega)^2 + \frac{1}{2} (\frac{1}{2} m_s r^2) \omega^2$$

$$0 + (3\text{kg})g(4\text{m}) = \frac{1}{2} (3\text{kg})(0.6\text{m})^2 \omega^2 + \frac{1}{4} (5\text{kg})(0.6\text{m})^2 \omega^2$$

$$\text{Solve for } \omega: 117.6 = 0.54\omega^2 + 0.45\omega^2$$

$$\omega = \sqrt{\frac{117.6}{0.54 + 0.45}} = 10.9 \text{ rad} / \text{s}$$