

# HOMework #1

1. Do problem 1.8 of the text.
2. Consider a star that is spherical. The star has a spot on it. The star has spherical coordinates  $(\vartheta, \varphi)$ , and the star is rotating. The center of the spot is at latitude  $\vartheta$  from the pole, and has azimuth  $\varphi = \omega t$ , for time  $t$  and angular rotation speed  $\omega$ .

Figure 1: Star geometry.

- a) Consider an observer in direction  $\vartheta = i$  for the  $i$  the viewing inclination and  $\varphi = 0$ . Both angles are fixed. The observer axis has coordinates  $(\theta, \alpha)$ . For a given inclination, prove that the spot center will NOT be occulted in its circuit around the star if it has latitude  $\vartheta < \pi/2 - i$ .
  - b) Use spherical trigonometry to find a relation  $\theta$  of the spot as a function of  $t, \vartheta, i$ .
  - c) Use this expression to determine  $d\theta/dt = \dot{\theta}$  as a function of  $\varphi$ .
3. Consider a planet orbiting a distant star. If the orbit is circular, the speed will be  $v(r) = v_0 \sqrt{R/r}$ , for  $R$  the stellar radius and  $r$  the orbital radius. In vector form  $\vec{v} = v(r) \hat{\varphi}$ , for  $\varphi$  the azimuth of the orbit about the star. An observer is in the direction  $(i, 0)$  along unit vector  $\hat{z}$ . Determine the projected velocity of the planet in the direction of the observer as a function of  $r, i, \varphi$ . (Use a dot product. Assume the planet is at latitude  $\vartheta = \pi/2$ .)

4. Problem 3.9 of the text.

5. Consider an object of luminosity  $L$  at distance  $r$  with apparent flux  $f$ .

- a) If you have an observational limiting detectable flux of  $f_{\min}$ , determine the maximum distance  $r_{\max}$  to which it can be seen in terms of luminosity.
- b) Let  $n(r)$  be the number density of these objects in space. If the objects are uniformly spread in direction, the number of objects out to some distance  $r$  will be

$$N = 4\pi \int_0^r n(r) r^2 dr.$$

With  $n(r) = n_0$  a constant,  $N = n_0 V$ , for spherical volume  $V$ . For limiting distance  $r_{\max}$ , find the maximum observable number of objects in the sky  $N_{\max}$ . Express this in terms of  $f_{\min}$ .

- c) Suppose you have limiting flux  $f_{\min}$ . Now consider making a distribution of the number of objects  $N$  in a survey with different flux cut-offs  $f$ . Show that

$$V/V_{\max} = (f_{\min}/f)^{3/2}.$$

This is a power-law relation. It is an expression of the volume to which you see for a given flux cut-off. It is a kind of cumulative distribution.

- d) Derive  $r/r_{\max}$  at which  $N/N_{\max} = 1/2$ . Suppose  $n(r)$  is not constant. How will this change the ratio  $r/r_{\max}$  where  $N/N_{\max} = 1/2$ ? Explain.
- e) Assume the constant density case again. Obviously, one object must be closest. If that object has a maximum flux  $f_{\max}$ , determine the average spacing between these objects.

6. Problem 5.1 of the text.

7. Problem 5.15 of the text.

8. Problem 6.7 of the text.