

HOMework #1

1. Do problem 1.8 of the text.
2. Consider a star that is spherical. The star has a spot on it. The star has spherical coordinates (ϑ, φ) , and the star is rotating. The center of the spot is at latitude ϑ from the pole, and has azimuth $\varphi = \omega t$, for time t and angular rotation speed ω .

Figure 1: Star geometry.

- a) Consider an observer in direction $\vartheta = i$ for the i the viewing inclination and $\varphi = 0$. Both angles are fixed. The observer axis has coordinates (ψ, α) . For a given inclination, prove that the spot center will NOT be occulted in its circuit around the star if it has latitude $\vartheta < \pi/2 - i$.
 - b) Use spherical trigonometry to find a relation ψ of the spot as a function of t, ϑ, i .
 - c) Use this expression to determine $d\psi/dt = \dot{\psi}$ as a function of φ .
3. Consider a planet orbiting a distant star. If the orbit is circular, the speed will be $v(r) = v_0 \sqrt{R/r}$, for R the stellar radius and r the orbital radius. In vector form $\vec{v} = v(r) \hat{\varphi}$, for φ the azimuth of the orbit about the star. An observer is in the direction $(i, 0)$ along unit vector \hat{z} . Determine the projected velocity of the planet in the direction of the observer as a function of r, i, φ . (Use a dot product. Assume the planet is at latitude $\vartheta = \pi/2$.)

4. Problem 3.9 of the text.

5. Consider an object of luminosity L at distance r with apparent flux f .

- a) If you have an observational limiting detectable flux of f_{\min} , determine the maximum distance r_{\max} to which it can be seen in terms of luminosity.
- b) Let $n(r)$ be the number density of these objects in space. If the objects are uniformly spread in direction, the number of objects out to some distance r will be

$$N = 4\pi \int_0^r n(r) r^2 dr.$$

With $n(r) = n_0$ a constant, $N = n_0 V$, for spherical volume V . For limiting distance r_{\max} , find the maximum observable number of objects in the sky N_{\max} . Express this in terms of f_{\min} .

- c) Suppose you have limiting flux f_{\min} . Now consider making a distribution of the number of objects N in a survey with different flux cut-offs f . Show that

$$V/V_{\max} = (f_{\min}/f)^{3/2}.$$

This is a power-law relation. It is an expression of the volume to which you see for a given flux cut-off. It is a kind of cumulative distribution.

- d) Derive r/r_{\max} at which $N/N_{\max} = 1/2$. Suppose $n(r)$ is not constant. How will this change the ratio r/r_{\max} where $N/N_{\max} = 1/2$? Explain.
- e) Assume the constant density case again. Obviously, one object must be closest. If that object has a maximum flux f_{\max} , determine the average spacing between these objects.

6. Problem 5.1 of the text.

7. Problem 5.15 of the text.

8. Problem 6.7 of the text.