

# HOMework #7

1. Problem 16.6 of the text.
2. Problem 16.10 of the text.
3. Problem 16.14 of the text.
4. Consider a spherically symmetric wind that is isothermal (i.e., constant temperature) and constant ionization. If the density of the wind is a power-law with  $n(r) \propto r^{-m}$ , for  $m$  some constant number, show that the resulting free-free radio spectrum is a power-law with  $f_\nu \propto \nu^{\alpha(m)}$ , where  $\alpha(m) = 2(2m - 3)/(2m - 1)$ . The free-free opacity at radio frequencies is  $\kappa_\nu \rho \propto \nu^{-2} n^2$ . First, assume the wind is quite optically thick. Then find  $R_\nu(m)$  for the condition that an observer sees to depth  $\tau_\nu \approx 1$ . Finally, use  $f_\nu \approx 4\pi^2 B_\nu R_\nu^2$ , with  $B_\nu \propto \nu^2$  in the radio band. Assume the Gaunt factor  $g_\nu$  is a constant. What is  $\alpha$  when  $m = 2$  and  $m \gg 1$ ?
5. Consider an emission line from a forbidden (i.e., fine structure) transition that forms in a wind.
  - a) Assume a two-level atom with lower level 1 and upper level 2. Let “r” be the ion state for an elemental species (e.g., NeIII would be  $r = 2$  for neon.) The level populations involve only collisional excitation from 1 to 2, collisional de-excitation from 2 to 1, and spontaneous decay from 2 to 1. Let  $A_{21}$  be the decay rate (per second),  $q_{12}$  be the collisional excitation volume rate (volume per second), and  $q_{21}$  be the collisional de-excitation volume rate. Derive  $n_2/n_r$  using  $n_r = n_1 + n_2$  and  $q_{12}n_1n_e = q_{21}n_2n_e + A_{21}n_2$ . Introduce parameters of the critical density  $n_c = A_{21}/q_{21}$  and  $\omega = q_{12}/q_{21}$ .
  - b) Determine  $n_2/n_r$  for  $n_e/n_c \gg 1$  and  $n_e/n_c \ll 1$ .
  - c) Adopting  $n_e = n_0 R^2/r^2$  for the density in the wind, where  $R$  is the stellar radius and  $n_0$  the density at the wind base, determine the critical radius where  $n_c = n_e(r_c)$ . This radius represents a transition from a high density region where collisional rates dominate the level population balance, to one where spontaneous decay dominates.
  - d) Forbidden lines are optically thin. The total flux of emission in the line is given by a volume integral over the emissivity. Introducing a kind of ion number fraction abundance  $\gamma_r = n_r/n_e$ , derive an expression for  $\gamma_r$  in terms of observed total line flux  $f_l$  using the following volume emissivity [ergs/sec/cm<sup>2</sup>/sr]:

$$j = \frac{1}{4\pi} n_2 A_{21} h\nu$$

and line flux [ergs/sec/cm<sup>2</sup>]

$$f_l = \frac{1}{D^2} \int_R^\infty 4\pi j r^2 dr$$

where  $D$  is the distance to the star. Note, it may be helpful to use a change of variable  $u = R/r$  to evaluate the integral.