

PHYS-2010: General Physics I
Course Lecture Notes
Section II

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Abstract

These class notes are designed for use of the instructor and students of the course **PHYS-2010: General Physics I** taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 8th Edition* (2009) textbook by Serway and Vuille.

II. Mathematical Techniques

A. Dimensional Analysis.

1. Always make sure that all terms in an equation have the same dimensions (*i.e.*, units).
2. Then try to reduce a parameter in an equation to a combination of the three basic concepts: **length** [**L**], **mass** [**M**], and **time** [**T**].
3. For example, the acceleration of a body in a gravitational field is proportional to the mass of the primary body and inversely proportional to the square of the distance:

$$a = G \frac{M}{r^2} ,$$

where G is a constant. From this formula, find the dimensions of G .

$$[a] = \mathbf{L T}^{-2} \quad [M] = \mathbf{M} \quad [r] = \mathbf{L} ,$$

where **L** represents *length*, **M** represents *mass*, and **T** represents *time*. Then

$$G = \frac{ar^2}{M} \implies [G] = \frac{[a] [r]^2}{[M]} = \frac{\mathbf{L T}^{-2} \mathbf{L}^2}{\mathbf{M}} = \mathbf{L}^3 \mathbf{M}^{-1} \mathbf{T}^{-2}$$

or the dimensions of G are $\text{m}^3/\text{kg}/\text{s}^2$.

4. When a symbol or variable has *square brackets* around it, this means: *what are the dimensions (i.e., units) of this symbol or variable?*

B. Algebra Review.

1. Cross multiplication: $mx = ny \iff \frac{x}{y} = \frac{n}{m}$.

2. Factoring: $y = mx + mb \iff y = m(x + b)$.

3. Powers & Roots:

a) $a \times a \times a \times a \times a \times a \times a = a^7$

or

$$a \times a \times \cdots (m - \text{times}) \cdots \times a = a^m$$

“ a ” is raised to the “ m^{th} ” power.

b) $a^{1/m} = \sqrt[m]{a} \implies$ “ m^{th} ” root of “ a .”

c) $a^0 \equiv 1$ (note that the “ \equiv ” symbol means “*defined to be*”).

d) $a^{-m} = \frac{1}{a^m}$.

e) $(ab)^m = a^m b^m$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} = a^m b^{-m}$.

f) $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$.

g) $(a^m)^n = a^{mn}$, $\sqrt[n]{a^m} = a^{m/n}$.

4. Exponentials and Logarithms:

$$y = a^x \quad (\text{base “}a\text{” to power “}x\text{”})$$

$$x = \log_a y \quad (\text{the exponent of “}a\text{” that yields “}y\text{”})$$

a) Product: $\log_a(xy) = \log_a x + \log_a y$.

b) Quotient: $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$.

c) Power: $\log_a(y^n) = n \log_a y$.

d) Two common bases:

i) Base 10 \Rightarrow **common logarithms:**

$$\log_a = \log_{10} \equiv \log$$

$$x = \log y \iff y = 10^x .$$

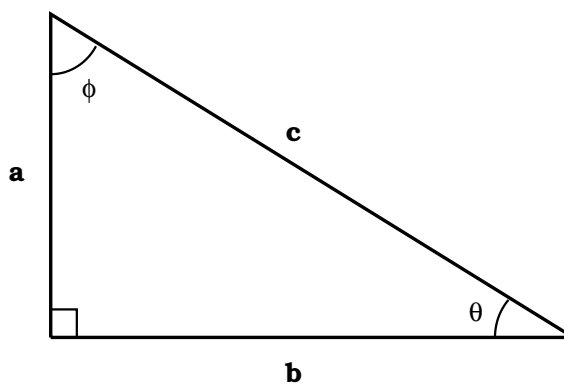
ii) Base $e = 2.71828\dots \Rightarrow$ **natural logarithms:**

$$\log_a = \log_e \equiv \ln$$

$$x = \ln y \iff y = e^x .$$

C. Basic Trigonometry.

1. Right-angle triangle relationships:

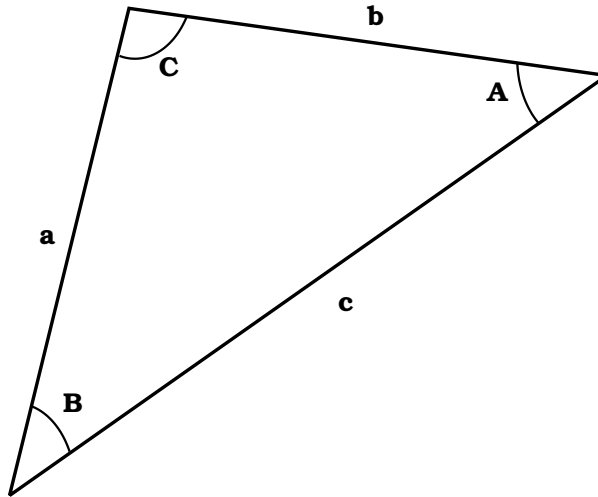


$$\sin \theta = \frac{a}{c}, \quad \cos \theta = \frac{b}{c}, \quad \tan \theta = \frac{a}{b} = \frac{\sin \theta}{\cos \theta}$$

$$a^2 + b^2 = c^2 \quad \text{or} \quad \sin^2 \theta + \cos^2 \theta = 1$$

$$\theta + \phi = 90^\circ = \frac{\pi}{2} \text{ radians.}$$

2. Generic triangle relationships:



a) Law of sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} .$$

b) Law of cosines:

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A , \\ b^2 &= a^2 + c^2 - 2ac \cos B , \\ c^2 &= a^2 + b^2 - 2ab \cos C . \end{aligned}$$

3. Additional useful trigonometric identities.

a) Angle-sum and angle-difference relations:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

b) Double-angle relations:

$$\begin{aligned}\sin 2\alpha &= 2 \sin \alpha \cos \alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha} \\ \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha \\ &= \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha} \\ \tan 2\alpha &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha}\end{aligned}$$

D. Scalars and Vectors.

1. A **scalar** has magnitude but no directional information (*e.g.*, c is a scalar).
 - a) 4 kg and 600 K are scalars.
 - b) 420 km/s is a scalar (*i.e.*, speed).
2. A **vector** has both magnitude and directional information (*e.g.*, \vec{c} is a vector).
 - a) 420 km/s to the NW is a vector (*i.e.*, velocity).
 - b) 420 km/s NW is not equal to 420 km/s SE!
 - c) Note that in these course notes I will always represent a vector with an arrow over the variable letter (*e.g.*, \vec{A}), whereas your textbook indicates a vector with a boldface letter (*e.g.*, \mathbf{A}).
 - d) Vector arithmetic will be described in §IV of the notes.