

PHYS-2010: General Physics I
Course Lecture Notes
Section VI

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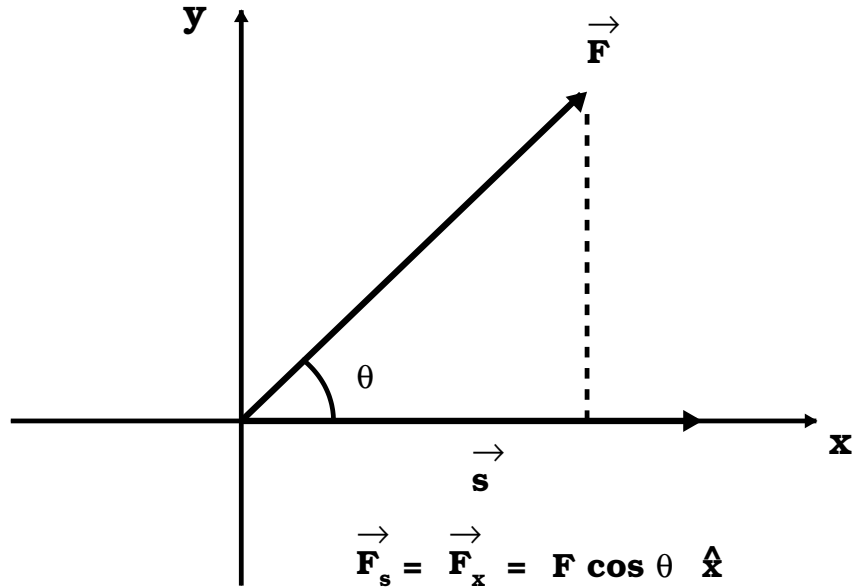
Abstract

These class notes are designed for use of the instructor and students of the course **PHYS-2010: General Physics I** taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 8th Edition* (2009) textbook by Serway and Vuille.

VI. Work and Energy

A. The Concept of Work.

1. **Work** is defined as the product of the component of the force along the direction of the displacement and the magnitude of the displacement.



- a) Component of F along displacement is $F \cos \theta$, where θ is the angle between \vec{F} and \vec{s} .
- b) The vector \vec{s} will be used sometimes to represent displacement $\implies \vec{x}, \vec{y}, \vec{z}, \vec{d}, \vec{r}, \vec{\ell}$, etc. will also be used from time to time.
- c) The unit for work in the SI unit system is
 - 1 Joule (J) \equiv 1 Newton-meter (N \cdot m).
 - i) In the cgs system: 1 erg \equiv 1 dyne-centimeter (dy \cdot cm) = 10^{-7} J.
 - ii) In the English system: 1 foot-pound (ft-lb) = 1.356 J.

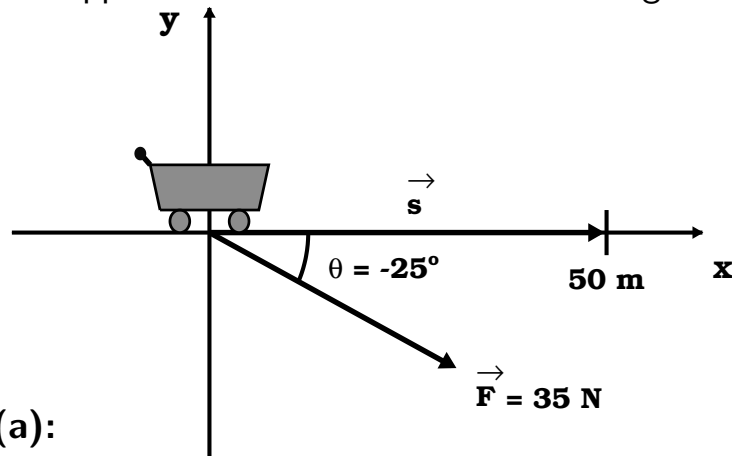
2. Mathematically, work is a scalar and expressed by:

$$W = \vec{F} \cdot \vec{s} = (F \cos \theta) s , \quad (\text{VI-1})$$

where $W =$ work, $F =$ force exerted on an object, $s =$ displacement over which F is acting, and $\theta =$ angle between \vec{F} and \vec{s} . Note that the ‘ \cdot ’ in Eq. (VI-1) is called the **dot-product**. The dot-product of two vectors always produces a scalar.

3. Work and energy are equivalent to each other and are measured in the same units \implies Joules in the SI system.

Example VI-1. Problem 5.4a (Page 153) from the Serway & Vuille textbook: A shopper in a supermarket pushes a cart with a force of 35 N directed at an angle of 25° downward from the horizontal. The force is just sufficient to overcome the various frictional forces, so the cart moves at a constant speed. (a) Find the work done by the shopper as she moves down a 50-m length of aisle.



Solution (a):

Using Eq. (VI-1), $W = (F \cos \theta) s$, along with $\theta = -25^\circ$, $F = 35$ N, and $s = 50$ m, we get

$$\begin{aligned} W &= [35 \text{ N} \cos(-25^\circ)] \cdot 50 \text{ m} = 32 \text{ N} \times 50 \text{ m} \\ &= \boxed{1600 \text{ J}} . \end{aligned}$$

B. The Work-Energy Theorem.

1. There are two main types of energy:

a) **Kinetic energy** (KE): energy of motion:

$$\boxed{\text{KE} \equiv \frac{1}{2} m v^2,} \quad (\text{VI-2})$$

where m is the mass of the object in motion and v is the velocity of motion.

b) **Potential energy** (PE): energy of position:

$$\boxed{\text{PE} \equiv m g y} \quad (\text{VI-3})$$

in a gravitational field, where m is the mass of the object, g is the acceleration due to gravity, and y is the height above some reference point (typically the ground).

c) Energies associated with different phenomenon can be classified into either of these two forms.

POTENTIAL

gravitational
chemical
elastic
electromagnetic
mass (nuclear reactions)

KINETIC

moving objects
heat
sound and other waves

2. The theorem: The net work done on an object by a net force acting on it is equal to the change in kinetic energy (final - initial) of the object.

$$\boxed{W_{\text{net}} = \text{KE}_f - \text{KE}_i = \Delta \text{KE} = \frac{1}{2} m (v^2 - v_o^2).} \quad (\text{VI-4})$$

Note that this equation is valid only if the work performed on an object does not cause any deformation of the object or cause a change the amount of heat present in the object.

- a) To prove this theorem, take Eq. (VI-1), and for simplicity sake, let's assume that F is being directed in the same direction as s (so $\theta = 0$) and finally let the total path length s lie along the x -axis so $s = x$. Using Newton's 2nd law (see Eq. V-1), we can rewrite Eq. (VI-1) as

$$W = F x = m a x .$$

- b) If we use the 1-D equation of motion given by Eq. (III-12), we see that

$$a x = \frac{1}{2} (v^2 - v_o^2) .$$

- c) As such, we can then write

$$W = m a x = \frac{1}{2} m (v^2 - v_o^2) = \frac{1}{2} m v^2 - \frac{1}{2} m v_o^2 = \text{KE}_f - \text{KE}_i ,$$

which proves the work-energy theorem.

C. Conservative and Nonconservative Forces.

1. A force is **conservative** if the work it does on an object moving between two points is independent of the path the object takes between the points.
 - a) The force of gravity is an example of a conservative force.
 - b) Remember that the force of gravity near the Earth's surface is given by Eq. (V-3):

$$F_g = m g .$$

- c) Let's now throw a ball upward a distance of $s = y_{\max} - y_o$. As the ball goes upward, the force of gravity points in the opposite direction of motion, therefore $\theta = 180^\circ$ in Eq.

(VI-1). Using this fact and equation above, Eq. (VI-1) can now be written as

$$\begin{aligned} W &= (F \cos \theta) s = (mg \cos 180^\circ) (y_{\max} - y_o) \\ &= -mg(y_{\max} - y_o) = mg(y_o - y_{\max}) \\ &= mgy_o - mgy_{\max} = PE_i - PE_f . \end{aligned}$$

d) As such, for gravity we can write

$$W_g = KE_f - KE_i = PE_i - PE_f = m g (y_i - y_f) . \quad (\text{VI-5})$$

e) We could make the same argument for any conservative force, therefore

$$\boxed{W_c = PE_i - PE_f = -\Delta PE ,} \quad (\text{VI-6})$$

\implies the work done by conservative force depends only on the change of potential energy \implies though, here, *initial minus final* (hence the negative sign in front of the ‘delta’ $[\Delta]$ symbol).

2. A force is **nonconservative** if the work done depends upon the path taken \implies leads to a dissipation of mechanical energy (friction is an example of a nonconservative force). For such forces, we must use the defining equation for work in order to calculate its value:

$$W_{nc} = (F_{nc} \cos \theta) s . \quad (\text{VI-7})$$

3. If both conservative and nonconservative forces cause an object to move, the work-energy theorem (*i.e.*, Eq. VI-4) becomes

$$W_{\text{net}} = W_c + W_{nc} = \Delta KE . \quad (\text{VI-8})$$

D. Conservation of Mechanical Energy: Total mechanical energy before an event = total mechanical energy after an event.

$$\boxed{KE_i + PE_i = KE_f + PE_f ,} \quad (\text{VI-9})$$

and for gravitational energy near the surface of the Earth:

$$\boxed{\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f .} \quad (\text{VI-10})$$

1. Potential energies come in a variety of forms:

a) Gravity (near a planetary surface):

$$\boxed{\text{PE}_g = mgy .} \quad (\text{VI-11})$$

b) Springs:

$$\boxed{\text{PE}_s = \frac{1}{2}kx^2 ,} \quad (\text{VI-12})$$

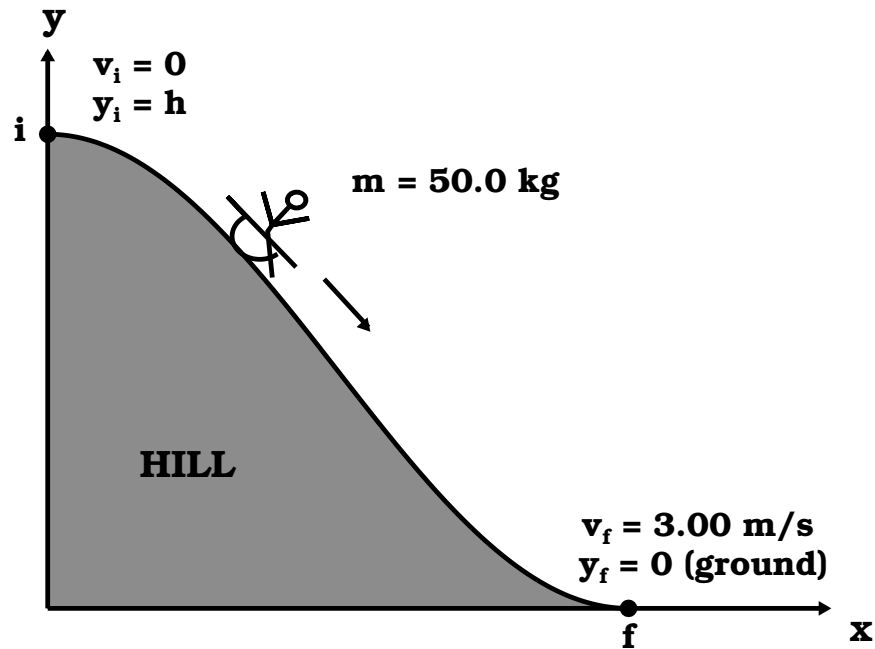
where k is the **spring constant** and x is the displacement between the compressed and uncompressed state of the spring.

2. The conservation of mechanical energy is only valid for conservative forces, *i.e.*, $W_{nc} = 0$. If nonconservative forces are not zero, then

$$\boxed{W_{nc} = (\text{KE}_f - \text{KE}_i) + (\text{PE}_f - \text{PE}_i) .} \quad (\text{VI-13})$$

\implies The work done by all nonconservative forces equals the change in kinetic energy plus the change in potential energy.

Example VI-2. A girl on a sled with a combined mass of 50.0-kg slides down a frictionless hill from rest. When she gets to the bottom of the hill, she is traveling at 3.00 m/s. How high is the hill?

**Solution:**

When most students see such a problem on an exam, their first instinct is to solve for the unknown using a 1-D or 2-D equations of motion. However, 1-D and 2-D equations of motion can only be used if the acceleration is constant over the path taken. Since we are not told whether the hill is smooth or has rises and dips in it, we cannot make this assumption here. As such, we should not use those equations here. Since the hill is said to be frictionless, the only force acting on the sled is the force of gravity, which is a conservative force. Due to this, we are free to use the conservation of mechanical energy here. Using Eq. (VI-10),

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f ,$$

we note that $m = 50.0 \text{ kg}$, $v_i = 0$ (sled starts from rest), $v_f = 3.00 \text{ m/s}$, $y_i = h$ (the height of the hill), and $y_f = 0$ (the ground). Solving for h we get

$$0 + mgh = \frac{1}{2}mv_f^2 + 0$$

$$\begin{aligned}
 gh &= \frac{1}{2}v_f^2 \\
 h &= \frac{v_f^2}{2g} = \frac{(3.00 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \frac{9.00 \text{ m}^2/\text{s}^2}{19.6 \text{ m/s}^2} \\
 &= \boxed{0.459 \text{ m} .}
 \end{aligned}$$

E. The Concept of Power.

1. **Power** is the time rate of doing work. Here, we shall define the average power as

$$\boxed{\bar{P} = \frac{W}{\Delta t} = \frac{F \Delta s}{\Delta t} = F \bar{v} .} \quad (\text{VI-14})$$

2. The unit of power in SI units is

$$\boxed{1 \text{ W (watt)} = 1 \text{ J/s} = 1 \text{ kg m}^2 / \text{s}^3 .}$$

- a) In the English system: 1 hp (horsepower) $\equiv 550 \frac{\text{ft}\cdot\text{lb}}{\text{s}} = 746 \text{ W}$.

- b) Energy in terms of power:

$$\begin{aligned}
 1 \text{ kilowatt-hour} &= 1 \text{ kWh} = (10^3 \text{ W})(3600 \text{ s}) = \\
 &= 3.600 \times 10^6 \text{ J} .
 \end{aligned}$$

Example VI-3. Water flows over a section of Niagara Falls at the rate of $1.2 \times 10^6 \text{ kg/s}$ and falls 50 m. How much power is generated by the falling water?

Solution:

Here we will use Eq. (VI-14), $\bar{P} = W/\Delta t$, in conjunction with

Eq. (VI-5), $W_g = PE_i - PE_f = mg(y_i - y_f)$ (since gravity is a conservative force). This gives a power of

$$\bar{P} = \frac{W_g}{\Delta t} = \frac{mg(y_i - y_f)}{\Delta t} = Rg(y_i - y_f) ,$$

where we have introduced a new term $R = m/\Delta t = 1.2 \times 10^6$ kg/s as the rate of mass flow over the falls. Using the values supplied for the position parameters, $y_f = 0$ and $y_i = 50$ m, we can solve for the power generated by the falls:

$$\begin{aligned} \bar{P} &= Rg(y_i - y_f) \\ &= (1.2 \times 10^6 \text{ kg/s}) (9.80 \text{ m/s}^2) (50 \text{ m} - 0) \\ &= \boxed{5.9 \times 10^8 \text{ W} = 590 \text{ MW} ,} \end{aligned}$$

where 'MW' represents *megawatts*.
