PHYS-2020: General Physics II
Course Lecture Notes
Section I

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Abstract

These class notes are designed for use of the instructor and students of the course PHYS-2020: General Physics II taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 9th Edition* (2012) textbook by Serway and Vuille.

I. Electric Forces & Electric Fields

A. Properties of Electric Charge.

1. Benjamin Franklin was the first to realize that there are *two types of electric charge*:

a) **Positive** charge: $+q$.

b) **Negative** charge: $-q$.

2. Like charges *repel* one another whereas unlike charges *attract* each other.

3. Electric charge is always conserved in any type of reaction or process.

4. In 1909, Robert Milliken discovered (via the **Millikan Oil-Drop Experiment**) that if an object is charged, its charge is always in multiples of the fundamental unit of charge, $\underline{e}$.

a) Charge is said to be **quantized**.

$\Longrightarrow \quad q = \pm e, \pm 2e, \pm 3e$, etc.

b) $e = 1.60219 \times 10^{-19}$ C (C $\equiv$ Coulomb, in the SI system) $= 4.80325 \times 10^{-10}$ esu (esu $\equiv$ electrostatic unit, in the cgs system).

c) Elementary particles:

i) Electron: $q = -e$.

ii) Proton: $q = +e$.

iii) Neutron: $q = 0$.

B. Insulators and Conductors.

1. **Conductors** are materials in which electric charges move freely (*i.e.*, they have low internal *resistance*).

   a) Copper.

   b) Aluminum.

   c) Silver.

2. **Insulators** are materials in which electric charges do <u>not</u> freely move (*i.e.*, they have high internal *resistance*).

   a) Glass.

   b) Rubber.

3. **Semiconductors** are materials that lie in between these other two $\Longrightarrow$ if controlled amounts of foreign atoms are added to semiconductors, their electrical properties can be changed by orders of magnitude.

   a) Silicon.

   b) Germanium.

4. The Earth can be considered to be an infinite reservoir of (or for) electrons.

   a) It can accept <u>or</u> supply an unlimited number of electrons.

   b) When a conductor is connected to the Earth (*e.g.*, conducting wire or copper pipe), it is said to be **grounded** $\Longrightarrow$ lightning rods.

5. An object can be charged in one of two ways:

a) **Conduction**: Charge exchange through contact.

i) If one charged object comes in contact with as second object, charge can move from the charged object to the uncharged object.

ii) Rubbing two different materials together, called **frictional work**, can produce negative charge on one object and a positive charge on the other.

b) **Induction**: Charge exchange with no contact.

i) Charge one object.

ii) This charge produces an **electric field** to form.

iii) This electric field can then induce charges to migrate on a second object $\Longrightarrow$ the second object **polarizes** (see Figure 15.4 in the textbook).

C. Coulomb's Law.

1. **Coulomb's law** states that two electric charges experience a force between them such that:

a) It is inversely proportional to the square of the separation r between the 2 particles along the line that joins them.

b) It is proportional to the product of the magnitudes of the charges, q_1 and q_2, on the two particles.

c) It is *attractive* if the charges are of opposite sign and *repulsive* if the charges have the same sign.

2. Mathematically, the Coulomb force F_e is

$$\boxed{\vec{F}_e = k_e \frac{|q_1|\,|q_2|}{r^2}\,\hat{r}\ .} \tag{I-1}$$

a) $k_e \equiv$ Coulomb's constant $= 8.9875 \times 10^9$ N m^2/C^2 (SI units) $= 1.000$ dyne cm^2/esu^2 (cgs units).

b) The sign of the electric force will depend upon the orientation of the defined coordinate system for a given problem and whether a given force is **repulsive** (*e.g.*, both q's are positive or both q's are negative) or **attractive** (*e.g.*, the charges are opposite in sign).

c) $\hat{r}$ is the unit vector in the radial direction.

3. Coulomb's law, like Newton's law of gravity ($F = Gm_1m_2/r^2$) is:

a) A **field force law** $\Longrightarrow$ there is no physical contact between the particles.

b) And an **inverse-square law** $\Longrightarrow$ the strength of the electric force falls off as the inverse of the distance squared.

4. If more than two charged particles exist in a system, then the Coulomb force exerted on one particle is the summation of all of the Coulomb forces between that particle and the rest of the particles in the ensemble:

$$\vec{F}_e = \sum_i^N F_i\,\hat{x} + \sum_j^N F_j\,\hat{y} + \sum_k^N F_k\,\hat{z}\ , \tag{I-2}$$

where F_i is the component Coulomb forces of all N particles in the x direction, F_j is the component forces in the y direction, and F_k is the component forces in the z direction. Eq. (I-2) is known as the **principle of superposition**.

Example I–1. An alpha particle (charge $+2.0e$) is sent at high speed toward a gold nucleus (charge $+79e$). What is the electrical force acting on the alpha particle when it is 2.0×10^{-14} m from the gold nucleus?

Solution:

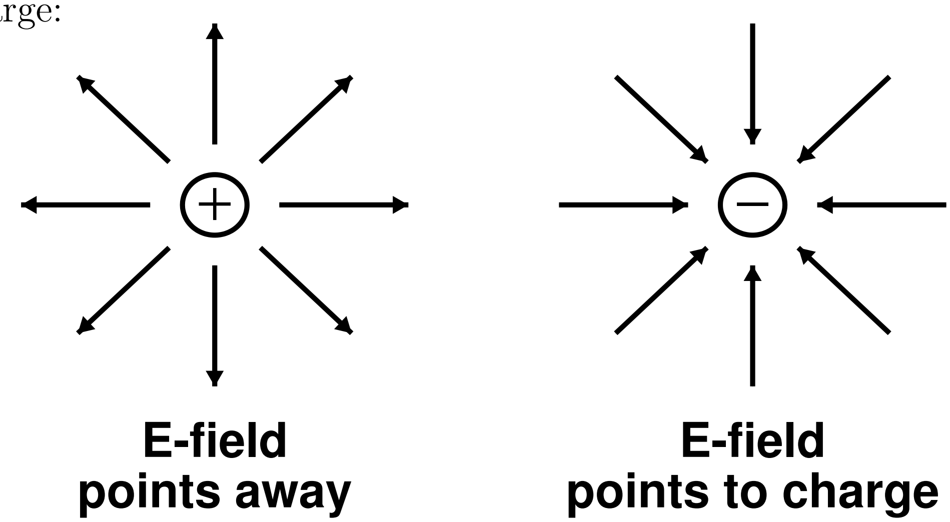
Use Coulomb's Law (*i.e.*, Eq. I-1) where $q_1 = +2e$, $q_2 = +79e$, $r = 2.0 \times 10^{-14}$ m, and $e = 1.60 \times 10^{-19}$ C:

$$\begin{aligned} |\vec{F}_e| &= k_e \frac{|q_1|\,|q_2|}{r^2} = k_e \frac{(2e)(79e)}{r^2} \\ &= \left(8.99 \times 10^9 \, \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right) \left[\frac{(2.0 \cdot 79)(1.60 \times 10^{-19}\ \mathrm{C})^2}{(2.0 \times 10^{-14}\ \mathrm{m})^2}\right] \\ &= \boxed{91\ \mathrm{N\ (repulsion).}} \end{aligned}$$

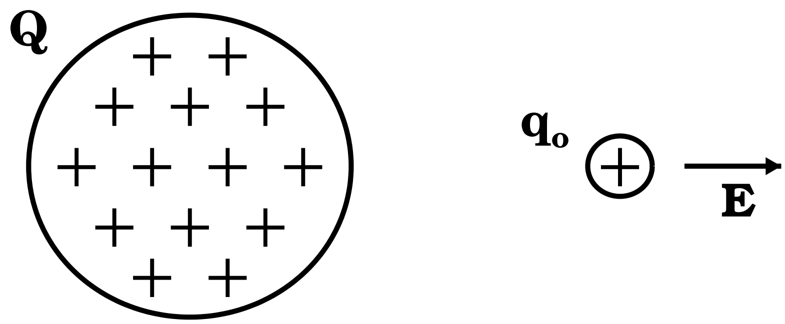
The Coulomb force is a repulsion force here since both particles have charges of the same sign.

D. The Electric Field.

1. An electric charge emits an **electric field** which always points *away* from a **positive** charge and points *towards* a **negative** charge:



a) Assume we take a *small* (*i.e.*, negligible) positive test charge $q_\circ$, and place it near a larger positively charged object, Q.



b) The direction of $\vec{E}$ (*i.e.*, the electric field) at a point is defined to be the direction of the electric force that would be exerted on a small (*i.e.*, negligible with respect to the charge on the larger object, $q_\circ \ll Q$) positive charge placed at that point.

c) Note that if $q_\circ$ starts to become comparable in size (in terms of charge) to Q, the electric field of Q will be altered by $q_\circ$.

2. Hence, the electric field is defined by the electric force exerted on a charged particle by another charged particle or object:

$$\boxed{\vec{E} = \frac{\vec{F}_e}{q_\circ}\ .} \tag{I-3}$$

a) Note that from this equation, E always points in the same direction that the force exerted on a positively charged particle $q_\circ$ by a charged object that gives rise to $\vec{E}$.

b) E is measured in N/C in SI units and dyne/esu in the cgs system.

c) The electric field $\vec{E}$ is analogous to surface gravity (*i.e.*, acceleration due to gravity) in Newton's Theory of Gravitation since we can write Newton's 2nd law as

$$\vec{g} = \frac{\vec{F}_g}{m} \ ,$$

where g is measured in N/kg (= m/s^2) and is nothing more than the gravitational force per unit mass (*i.e.*, the strength of the gravitational field). Likewise, the electric field is the electrical force per unit charge.

3. Let the object that produces an E-field be given a charge labeled with q. Then a test charge $q_\circ$ a distance r from q experiences a force from q described by Coulomb's law (*i.e.*, Eq. I-1):

$$\vec{F}_e = k_e \frac{|q|\,|q_\circ|}{r^2}\,\hat{r} \ .$$

Plugging this into Eq. (I-3) gives a second equation that describes the E-field:

$$\boxed{\vec{E} = k_e \frac{|q|}{r^2}\,\hat{r} \ .} \qquad \text{(I-4)}$$

a) If q is positive (*i.e.*, $q > 0$), then E is positive and points radially away from the charge.

b) If q is negative (*i.e.*, $q < 0$), then E is negative and points radially inward towards the charge.

Example I–2. An electron with a speed of 3.00×10^6 m/s moves into a uniform electric field of 1000 N/C. The field is parallel to the electron's motion. How far does the electron travel before it is brought to rest?

Solution:

When the electron, with mass $m_e = 9.11 \times 10^{-31}$ kg, enters the electric field, it experiences a retarding force given by Eq. (I-3):

$$F = -eE \ ,$$

negative since the E-field slows the electron. Since this force slows the electron, it produces a deceleration. Using Newton's 2nd law we can write

$$\begin{aligned} a &= \frac{F}{m_e} = \frac{-eE}{m_e} = -\frac{(1.60 \times 10^{-19}\ \text{C})(1000\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}} \\ &= -1.76 \times 10^{14}\ \text{m/s}^2 \ . \end{aligned}$$

Using one of the 1-D equations of motion from General Physics I, we can solve for the distance that the electron travels before coming to a stop, Δx. We have $v = 0$ (e^- comes to rest), $v_\circ = 3.00 \times 10^6$ m/s, and a has been calculated above, giving

$$\begin{aligned} v^2 &= v_\circ^2 + 2a\,\Delta x \\ \Delta x &= \frac{v^2 - v_\circ^2}{2a} \\ &= \frac{0 - (3.00 \times 10^6\ \text{m/s})^2}{2.0(-1.76 \times 10^{14}\ \text{m/s}^2)} \\ &= 0.0256\ \text{m} = \boxed{2.56\ \text{cm}\ .} \end{aligned}$$

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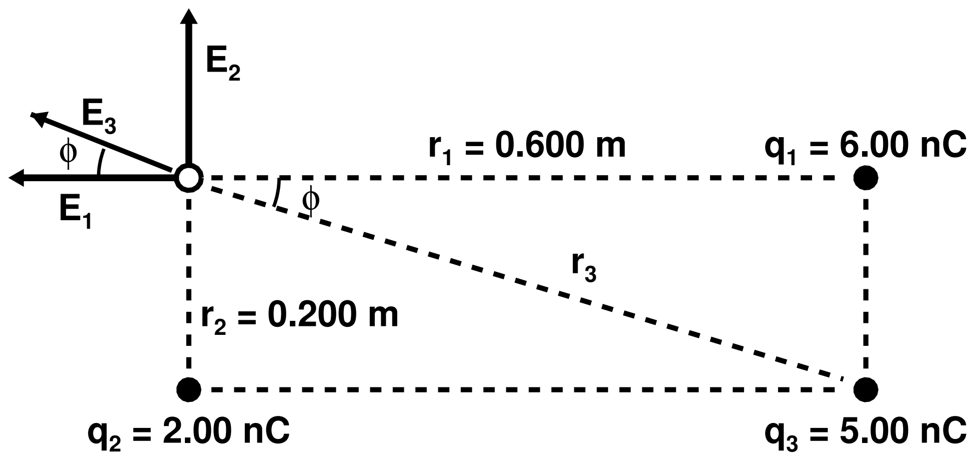
4. When summing E-fields from multiple charges, it is best to draw a vector diagram first at a point where the E-field is to be determined. Draw the E-field vector of each charge in the direction as dictated by the rules above. Then vectorially add the individual E-fields together, letting the diagram determine the sign of the

E-field (typically + to the right and + upwards). Then add using the absolute value of the charge:

$$E = \sum_{1}^{N} (\pm) \frac{k_e |q|}{r^2} \ , \qquad \text{(I-5)}$$

where either + or − is selected based upon the direction of the E-field vector (see Example I-3 below).

Example I–3. Positive charges are situated at three corners of a rectangle as shown in the figure below. Find the electric field at the fourth corner.



Solution:

From the geometry of the rectangle as shown in the figure above, we have

$$r_3^2 = r_1^2 + r_2^2 = (0.600 \text{ m})^2 + (0.200 \text{ m})^2 = 0.400 \text{ m}^2 \ .$$

and

$$\phi = \tan^{-1}\left(\frac{r_2}{r_1}\right) = \tan^{-1}\left(\frac{0.200 \text{ m}}{0.600 \text{ m}}\right) = 18.4^\circ \ .$$

The components of the individual E-field vectors are

$$\begin{aligned} \vec{E}_1 &= -E_{1x}\,\hat{x} = -E_1\,\hat{x} \\ \vec{E}_2 &= +E_{2y}\,\hat{y} = +E_2\,\hat{y} \\ \vec{E}_3 &= -E_{3x}\,\hat{x} + E_{3y}\,\hat{y} \\ &= -E_3 \cos\phi\,\hat{x} + E_3 \sin\phi\,\hat{y} \ . \end{aligned}$$

The resultant E-field at the vacant corner is

$$\vec{E}_R = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

with the resultant component vectors of

$$\begin{aligned}
E_x &= \sum_{i=1}^{3} E_{ix} = -E_{1x} + 0 - E_{3x} = -E_1 - E_3 \cos\phi \\
&= -\frac{k_e|q_1|}{r_1^2} - \frac{k_e|q_3|}{r_3^2}\cos\phi \\
&= -\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{6.00 \times 10^{-9}\ \text{C}}{(0.600\ \text{m})^2} + \right. \\
&\qquad \left. \frac{5.00 \times 10^{-9}\ \text{C}}{0.400\ \text{m}^2}\cos 18.4^\circ\right] \\
&= -256\ \text{N/C}
\end{aligned}$$

$$\begin{aligned}
E_y &= \sum_{j=1}^{3} E_{jy} = 0 + E_{2y} + E_{3y} = E_2 + E_3 \sin\phi \\
&= \frac{k_e|q_2|}{r_2^2} + \frac{k_e|q_3|}{r_3^2}\sin\phi \\
&= \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{3.00 \times 10^{-9}\ \text{C}}{(0.200\ \text{m})^2} + \right. \\
&\qquad \left. \frac{5.00 \times 10^{-9}\ \text{C}}{0.400\ \text{m}^2}\sin 18.4^\circ\right] \\
&= 710\ \text{N/C}
\end{aligned}$$

Thus,

$$E_R = \sqrt{(E_x)^2 + (E_y)^2} = 755\ \text{N/C}\ .$$

With $E_x < 0$ and $E_y > 0$, the resultant E-field lies in the second quadrant with an angle of

$$\theta_{(-)} = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{710\ \text{N/C}}{-256\ \text{N/C}}\right) = -70^\circ\ .$$

However, this is with respect to the $-x$ axis. θ (with respect to the $+x$ axis) is then $\theta = 180^\circ + \theta_{(-)} = 180^\circ - 70^\circ = 110^\circ$. Hence, we can express the answer in one of two ways:

$$\vec{E}_R = -(256 \text{ N/C})\,\hat{x} + (710 \text{ N/C})\,\hat{y}$$
or
$$\vec{E}_R = 755 \text{ N/C} \;\text{ at } 110^\circ \text{ counterclockwise from the } +x \text{ axis.}$$

E. Electric Field Lines.

1. Drawing electric field lines:

a) The electric field vector, $\vec{E}$, is tangent to the electric field lines at each point.

b) The number of lines per unit area through a surface $\perp$ to the lines is proportional to the strength of the electric field in a given region.

i) $\vec{E}$ is large when the field lines are close together.

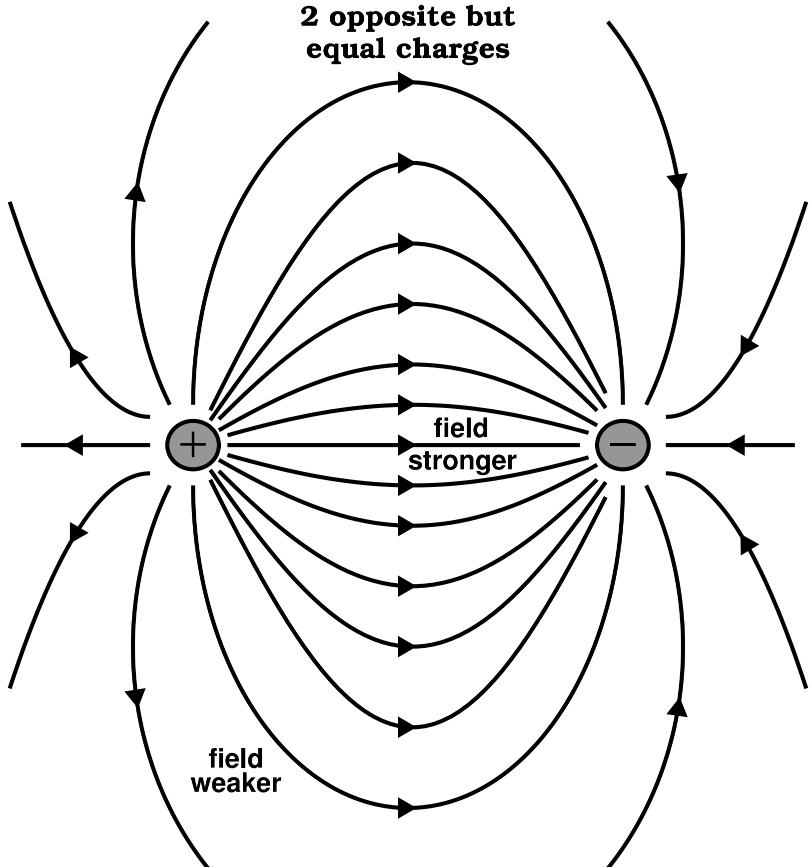
ii) $\vec{E}$ is small when the field lines are far apart.

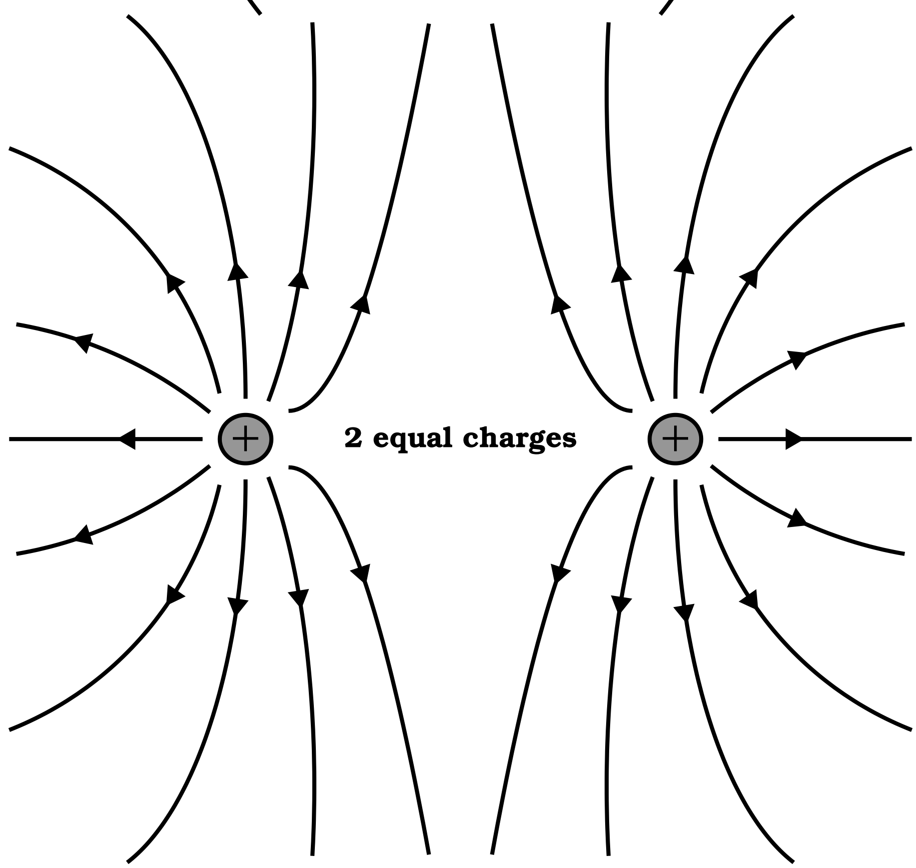
2. For a system of charged particles, the following rules apply:

a) The lines must begin on positive charges (or at infinity) and must terminate on negative charges (or, in the case of excess charge, at infinity).

b) The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge (*e.g.*, if a charge of $+q$ has n-lines per unit volume leaving the charge, a charge of $-2q$ will have $2n$-lines per unit volume approaching the charge — see Ex. I-4).

c) No 2 field lines can cross each other.


2 opposite but
equal charges
field
stronger
field
weaker


2 equal charges

Example I–4. **Problem 15.30 (Page 544) from the Serway & Vuille textbook**: Figure P15.30 shows the electric field lines for two point charges separated by a small distance. (a) Determine the ratio q_1/q_2. (b) What are the signs of q_1 and q_2?

Solution (a & b):

The magnitude of q_2 is 3 times the magnitude of q_1 since 3 times as many lines emerge from q_2 as enter q_1. Hence, $|q_2| = 3|q_1|$. Since the field lines are emerging from q_2, it must have a positive charge, and since they terminate on q_1, it must have a negative charge. Thus,

$$\textbf{(a)} \qquad \boxed{\frac{q_1}{q_2} = -\frac{1}{3}\ ,}$$

and

$$\textbf{(b)} \qquad \boxed{q_1 < 0, \quad q_2 > 0\ .}$$

F. Conductors in Electrostatic Equilibrium.

1. A good conductor contains electrons that are not bound to any one atom $\Longrightarrow$ free to move about the material.

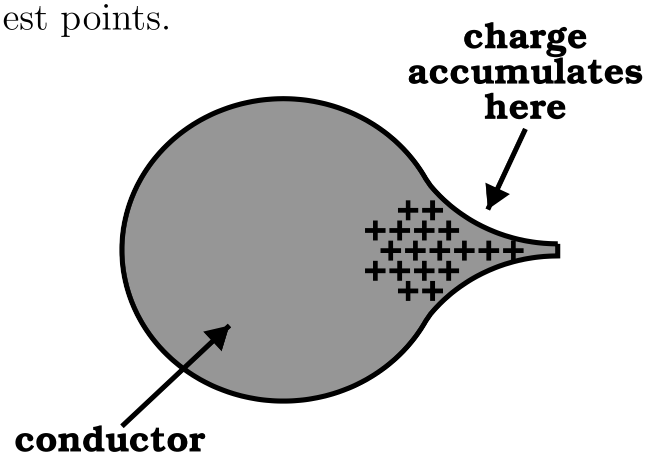
2. **Electrostatic Equilibrium:** No net motion of charge occurs within a conductor. The following is true for such a conductor:

a) The electric field is zero everywhere inside the conductor.

b) Any excess charge on an isolated conductor resides entirely on its surface.

c) The electric field just outside a charged conductor is $\perp$ to the conductor's surface.

d) On an irregular shaped conductor, charge accumulates at its sharpest points.



G. The Electric Flux and Gauss's Law.

1. The **electric flux** is a measure of the number of E-field lines that crosses a given area.

a) An E-field whose lines penetrate a cross-sectional (or surface) area $A \perp$ to A has an electric flux Φ_E given by

$$\Phi_E = E\,A\ . \tag{I-6}$$

b) However, if the E-field lines lie at an angle θ with respect to the normal line of area A (see Figure 15.25 in the textbook), the electric flux is given by the more general formula:

$$\boxed{\Phi_E = \vec{E} \cdot \vec{A} = E\,A\,\cos\theta\ ,} \tag{I-7}$$

where the '$\cdot$' is called the **dot product** operation and $\vec{A}$ has magnitude A (the total cross-sectional or surface area) and direction given by the normal line of the area. The angle θ is the angle between that normal line and the E-field direction.

c) When the area is constructed such that a closed surface is formed, we shall adopt the convention that the flux lines passing <u>into</u> the interior of the volume are *negative* and those passing <u>out</u> of the interior of the volume are *positive*.

2. One of the most important law's in all of electromagnetism is Gauss's law:

a) If we use a sphere for our enclosing volume, the sphere has a surface area of $A = 4\pi r^2$. If we place charge q at the center of this sphere, then the field lines will always be $\perp$ to the surface of the sphere since they point radially outward. Using Eq. (I-4) in Eq. (I-6), we can write the electric flux as

$$\Phi_E = E\,A = k_e \frac{q}{r^2}\left(4\pi r^2\right) = 4\pi k_e q \ ,$$

independent of the distance from the charge!

b) In electromagnetism, there is a constant related to the Coulomb constant called the **permittivity of free space** $\epsilon_\circ$. This constant is given by

$$\epsilon_\circ = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12}\, \frac{\mathrm{C}^2}{\mathrm{N \cdot m}^2} \ . \qquad \text{(I-8)}$$

c) Using this constant in our flux equation above, we can write the electric flux as

$$\Phi_E = 4\pi k_e q = \frac{q}{\epsilon_\circ} \ .$$

d) Using calculus, we could show that this simple result is true for any closed surface (even non-symmetrical ones) that surrounds any charge $q \Longrightarrow$ such a surface is referred to as a *gaussian surface*.

e) This bit of mathematics was first worked out by Gauss and is therefore called **Gauss's law**.

i) In words, this law states: **The electric flux through any closed surface is equal to the net charge Q inside the surface divided by the permittivity of free space $\epsilon_\circ$.**

ii) Mathematically, it is given by

$$\boxed{\Phi_E = \frac{Q_{\text{inside}}}{\epsilon_\circ}\ .} \tag{I-9}$$

Example I–5. **Problem 15.41 (Page 545) from the Serway & Vuille textbook**: An electric field of intensity 3.50 kN/C is applied along the x axis. Calculate the electric flux through a rectangular plane 0.350 m wide and 0.700 m long if (a) the plane is parallel to the yz plane; (b) the plane is parallel to the xy plane; and (c) the plane contains the y axis and its normal makes an angle of 40.0° with the x axis.

Solution (a):

The flux through an area is given by Eq. (I-7), where θ is the angle between the direction of the field E and the normal line of area A. The magnitude of the area of the plane is $A = (0.350 \text{ m})(0.700 \text{ m}) = 0.245 \text{ m}^2$. When the plane is parallel to the yz plane, the normal line of the area lies parallel to the x axis, so $\theta = 0$, and the flux is

$$\begin{aligned}\Phi_E = EA\cos\theta &= (3.50 \times 10^3 \text{ N/C})(0.245 \text{ m}^2)\cos 0^\circ \\ &= \boxed{858 \text{ N} \cdot \text{m}^2/\text{C}\ .}\end{aligned}$$

Solution (b):

When the plane is parallel to the x axis (as it must be when it is parallel to the xy plane), the normal line of the area is at a right angle to the x axis (and hence E field), so $\theta = 90^\circ$ and

$$\Phi_E = EA\cos\theta = (3.50 \times 10^3 \text{ N/C})(0.245 \text{ m}^2)\cos 90^\circ = \boxed{0}\ .$$

Solution (c):

Since the E field is along the x axis and the normal of the area make an angle of 40.0° with respect to the x axis, $\theta = 40.0^\circ$, so

$$\begin{aligned}\Phi_E = EA\cos\theta &= (3.50 \times 10^3 \text{ N/C})(0.245 \text{ m}^2)\cos 40.0^\circ \\ &= \boxed{657 \text{ N} \cdot \text{m}^2/\text{C}}\ .\end{aligned}$$