

**PHYS-2020: General Physics II**  
**Course Lecture Notes**  
**Section III**

Dr. Donald G. Luttermoser  
East Tennessee State University

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## **Abstract**

These class notes are designed for use of the instructor and students of the course **PHYS-2020: General Physics II** taught by Dr. Donald Luttermoser at East Tennessee State University. These notes make reference to the *College Physics, 9th Edition* (2012) textbook by Serway and Vuille.

### III. Current & Resistance

#### A. Electric Current.

1. **Current** is defined as the rate at which charge flows through a surface.

- a) Mathematically:

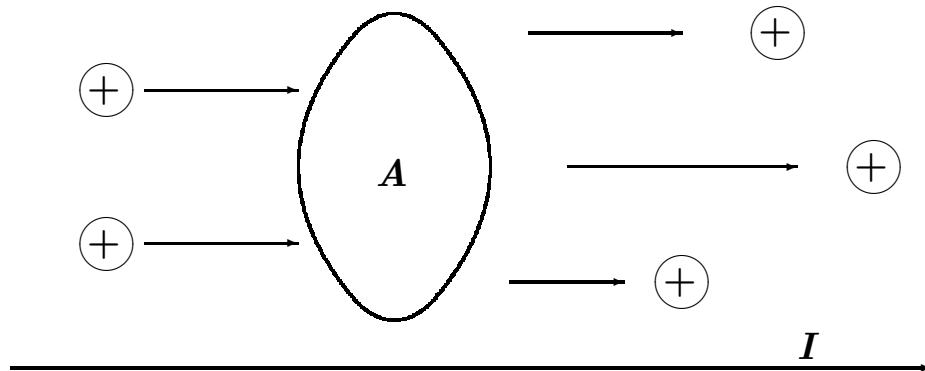
$$\boxed{I = \frac{\Delta Q}{\Delta t}}, \quad (\text{III-1})$$

where  $I$  is the current,  $\Delta Q$  is the amount of charge passing through an area of wire, and  $\Delta t$  is the time interval in which  $\Delta Q$  is measured.

- b) Current is measured in **amperes** in the SI unit system:

$$1 \text{ A} \equiv 1 \text{ C/s} . \quad (\text{III-2})$$

2. The direction of current is defined to be the direction at which a positive charge would flow through a wire.



- a) In metals, it is electrons that flow and not positive charges  $\implies$  the electrons flow in the *opposite* direction of the current!
- b) Moving charge (whether positive or negative) through a conductor is known as a mobile **charge carrier**.

3. Electrons flow in the opposite direction of the  $\vec{E}$ -field.

a) As an electron (or any charged particle) moves through a conductor, it collides with atoms (and/or molecules) in the conductor  $\implies$  causes a *zigzag* motion through the conductor.

b) The amount of charge passing through a wire can be determined as follows:

i) Let  $A$  be the cross-sectional area of a wire and  $\Delta x$  be a small slice along the length of the wire.

ii) The volume of this small segment of the wire is then  $V = A \Delta x$  (**note** that  $V$  here is volume not voltage).

iii) Let  $N$  be the number of charge carriers contained in this volume and  $q$  be the charge per carrier. Then,

$$n = \frac{N}{V} = \frac{N}{A \Delta x} ,$$

represents the number of carriers per unit volume.

iv) The total charge contained in this volume is thus

$$\Delta Q = Nq = (n A \Delta x) q . \quad (\text{III-3})$$

c) Although the electron makes a *zigzag* path through the wire, *on average*, it continues to move down the electric field (remember in the opposite sense) at an average speed called the **drift speed**  $v_d$ :

$$v_d = \frac{\Delta x}{\Delta t} \implies \Delta x = v_d \Delta t .$$

We can then substitute this into Eq. (III-3) giving

$$\Delta Q = (n A v_d \Delta t) q . \quad (\text{III-4})$$

d) Dividing both sides by  $\Delta t$  gives

$$\frac{\Delta Q}{\Delta t} = I = n q v_d A \quad (\text{III-5})$$

or

$$v_d = \frac{I}{n q A} .$$

Since we normally talk about electrons in a metal wire, we can rewrite this as

$$v_d = \frac{I}{n |e| A} . \quad (\text{III-6})$$

e) If no current exists in a conductor, the electric field is zero inside the conductor. However, if current exists, an electric field exists inside the conductor (due to Maxwell's laws — see §IX of the notes).

**Example III-1. Problem 17.7 (Page 611) from the Serway & Vuille textbook:** A 200-km long high-voltage transmission line 2.0 cm in diameter carries a steady current of 1000 A. If the conductor is copper with a free charge density of  $8.5 \times 10^{28}$  electrons per cubic meter, how long (in years) does it take one electron to travel the full length of the cable? *Added question:* How long would it take a photon to travel the same distance?

**Solution:**

The drift speed of electrons in the line is (from Eq. III-6)

$$v_d = \frac{I}{n q A} = \frac{I}{n |e| (\pi D^2/4)} ,$$

where  $I = 1000 \text{ A} = 1000 \text{ C/s}$  is the current,  $n = 8.5 \times 10^{28}$  electrons/ $\text{m}^3$  is the charge carrier density,  $|e| = 1.60 \times 10^{-19} \text{ C}$  is the charge per electron, and  $D = 2.0 \text{ cm} = 0.020 \text{ m}$  is the diameter of the wire. Solving for the drift velocity gives

$$v_d = \frac{4(1000 \text{ C/s})}{(8.5 \times 10^{28} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})\pi(0.020 \text{ m})^2} = 2.3 \times 10^{-4} \text{ m/s} .$$

The time to travel the length  $L$  of the 200 km =  $2.00 \times 10^5$  m wire is then

$$\Delta t = \frac{L}{v_d} = \frac{2.00 \times 10^5 \text{ m}}{2.34 \times 10^{-4} \text{ m/s}} \left( \frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}} \right) = \boxed{27.1 \text{ yr .}}$$

For the additional question, since light (*i.e.*, photons) travels at a speed of  $c = 3.00 \times 10^8$  m/s, the time it takes a photon to travel this length is

$$\Delta t = \frac{L}{c} = \frac{2.00 \times 10^5 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 6.67 \times 10^{-4} \text{ s} = \boxed{667 \mu\text{s} .}$$

Note that even though an individual electron in the flow of electricity takes a long time to travel through the wire, the  $E$ -field inside the wire propagates along the wire at the speed of light.

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## B. Resistance and Ohm's Law.

1. **Resistance** is the ratio of voltage difference to current:

$$R \equiv \frac{\Delta V}{I} . \quad (\text{III-7})$$


- a) Measured in **ohms**:

$$\Omega = \frac{\text{V}}{\text{A}} = \frac{\text{volts}}{\text{amperes}} . \quad (\text{III-8})$$

- b) Resistance measures how hard it is for electrons (or any charged particle) to flow through material  $\implies$  it essentially measures the number of internal collisions an electron (or any charged particle) has in a circuit.
- c) The higher the resistance, the more collisions with internal atoms/molecules that make up the wire or resistor. The larger the number of collisions per second, the larger the amount of heat generated in the wire or resistor.

- 2. Ohm's Law:** Resistance that remains constant over a wide range of applied voltage differences such that the voltage difference is linearly dependent on current:

$$\boxed{\Delta V = I R .} \quad (\text{III-9})$$

- a) Materials that obey this law are called *ohmic*  $\implies$  conductors are ohmic.
- b) Materials that do not obey this law are called *non-ohmic*  $\implies$  semiconductors are non-ohmic.
- 3.** A **resistor** is a simple circuit element that provides a specific resistance in an electric circuit. Its symbol is 

### C. Resistivity.

- 1.** Since resistance is related to the number of collisions an electron has with atoms/molecules of the wire, we can describe resistance in terms of the geometric properties of the conductor and the composition of the conductor (*i.e.*, ohmic material):

$$\boxed{R = \rho \frac{L}{A} ,} \quad (\text{III-10})$$

where  $L$  is the length of the conductor,  $A$  is the cross-sectional area of the conductor (both of these are the geometric part), and  $\rho$  is the resistivity of the material (which is related to the composition of the material — see Table 17.1 in the textbook).

- 2.** Electric conductors have low resistivity, insulators have high resistivity.
- 3.** Resistivities can change with temperature (see §III.D).

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**Example III-2. Problem 17.17 (Page 611) from the Serway & Vuille textbook:** A wire 50.0 m long and 2.00 mm in diameter is connected to a source with a potential difference of 9.11 V, and the current is found to be 36.0 A. Assume a temperature of 20°C and, using Table 17.1, identify the metal of the wire.

**Solution:**

From Ohm's law (Eq. III-9) we can calculate the resistance of the wire to be

$$R = \frac{\Delta V}{I} = \frac{9.11 \text{ V}}{36.0 \text{ A}} = 0.253 \Omega .$$

The diameter of the wire is  $D = 2.00 \text{ mm} = 2.00 \times 10^{-3} \text{ m}$ , so the resistivity (using Eq. III-10) of the metal is

$$\begin{aligned} \rho &= \frac{RA}{L} = \frac{R(\pi D^2/4)}{L} = \frac{(0.253 \Omega)\pi(2.00 \times 10^{-3} \text{ m})^2}{4(50.0 \text{ m})} \\ &= 1.59 \times 10^{-8} \Omega \cdot \text{m} . \end{aligned}$$

Comparing this value to those listed in Table 17.1 of the textbook, the metal must be 

silver.
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#### D. Temperature Variation of Resistance.

1. For most metals, resistivity increases in an approximate linear fashion with temperature:

$$\rho = \rho_0 [1 + \alpha (T - T_0)] . \quad (\text{III-11})$$

- a)  $\rho$  is the resistivity at some temperature  $T$ .
- b)  $\rho_0$  is the resistivity at some temperature  $T_0$ .
- c)  $\alpha$  is the **temperature coefficient of resistivity** (see Table 17.1).

2. If a wire is of constant cross-sectional area  $A$  and length  $L$  with respect to change in temperature, we can write

$$R = R_o [1 + \alpha (T - T_o)] . \quad (\text{III-12})$$

3. Thermometers that measure temperature from resistance in their circuit are called **thermistors** (also called **thermocouples**).

**Example III-3.** At  $40.0^\circ\text{C}$ , the resistance of a segment of gold wire is  $100.0 \Omega$ . When the wire is placed in a liquid bath, the resistance decreases to  $97.0 \Omega$ . What is the temperature of the bath? (*Hint*: First determine the resistance of the gold wire at room temperature.)

**Solution:**

If  $R = 100.0 \Omega$  at  $T = 40.0^\circ\text{C}$ , then the resistance at  $T_o = 20.0^\circ\text{C}$  (*i.e.*, room temperature) can be found with Eq. (III-12). The thermal resistance coefficient for gold at  $20.0^\circ\text{C}$  is  $\alpha = 3.40 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$ , so

$$\begin{aligned} R_o &= \frac{R}{1 + \alpha(T - T_o)} \\ &= \frac{100.0 \Omega}{1 + (3.40 \times 10^{-3} \text{ }^\circ\text{C}^{-1})(40.0 \text{ }^\circ\text{C} - 20.0 \text{ }^\circ\text{C})} \\ &= 93.6 \Omega . \end{aligned}$$

Now we once again use Eq. (III-12) to determine the temperature of the bath. After a little algebra, we get

$$\begin{aligned} T &= T_o + \frac{R - R_o}{\alpha R_o} = 20.0 \text{ }^\circ\text{C} + \frac{97.0 \Omega - 93.6 \Omega}{(3.40 \times 10^{-3} \text{ }^\circ\text{C}^{-1})(93.6 \Omega)} \\ &= \boxed{30.6 \text{ }^\circ\text{C} .} \end{aligned}$$

### E. Superconductors.

1. There is a class of metals and compounds whose resistance virtually goes to zero below a certain temperature,  $T_c$ , called the **critical temperature** (see Figure 17.11 in the textbook).
2. These materials are known as **superconductors**.
3. Once current has been set up in a superconductor, it persists without any applied voltage (since  $R = 0$ ).

### F. Electrical Energy and Power.

1. A battery can deliver **power** to an electric circuit. Power (as covered in General Physics I) is the amount of work exerted over an interval of time:

$$\mathcal{P} = \frac{W}{\Delta t} . \quad (\text{III-13})$$

2. Since work is equal to the change of potential energy (see Eq. II-3), we can write

$$W = \Delta\text{PE} = q \Delta V$$

or

$$\mathcal{P} = q \frac{\Delta V}{\Delta t} . \quad (\text{III-14})$$

3. If we have a current of charges  $\Delta q$  across a voltage difference  $\Delta V$ , we can rewrite Eq. (III-14) as

$$\mathcal{P} = \Delta q \frac{\Delta V}{\Delta t} = \Delta V \frac{\Delta q}{\Delta t} = (\Delta V) I$$

or

$$\boxed{\mathcal{P} = I \Delta V .} \quad (\text{III-15})$$

As we can see from this equation, the unit for power ( $W = \text{watt}$ ) must be equal to amperes times volts, or

$$1 \text{ W} = \text{A} \cdot \text{V} . \quad (\text{III-16})$$

4. Using Ohm's law (Eq. III-9), we can also express Eq. (III-15) as

$$\mathcal{P} = I^2 R = \frac{(\Delta V)^2}{R} . \quad (\text{III-17})$$

- a) The power delivered to a conductor of resistance  $R$  is often referred to as an  $I^2 R$  loss.
- b) Note that Eq. (III-17) applies only to resistors (and other ohmic devices) and not to non-ohmic devices like light bulbs and diodes.
- c) Electric companies measure the amount of power you have used over the billing cycle (= total energy used). The units used by the electric company are **kilowatt-hour**:

$$1 \text{ kWh} = (10^3 \text{ W})(3600 \text{ s}) = 3.60 \times 10^6 \text{ J} . \quad (\text{III-18})$$

**Example III-4. Problem 17.34 (Page 612) from the Serway & Vuille textbook:** If the electrical energy costs 12 cents per kilowatt-hour, how much does it cost to (a) burn a 100-W lightbulb for 24 hr, and (b) operate an electric oven for 5.0 hr if it carries a current of 20.0 A at 220 V?

**Solution (a):**

The energy produced by the 100-W lightbulb in one day (24 hr) is

$$E = \mathcal{P} \cdot \Delta t = (100 \text{ W})(24 \text{ hr}) = (0.100 \text{ kW})(24 \text{ hr}) = 2.40 \text{ kWh} ,$$

where kWh = kilowatt-hour. At a rate of  $\mathcal{R} = 12\text{-cents}$  (= \$0.12)

per kWh, the cost of this energy is

$$\begin{aligned}\text{cost} &= E \cdot \mathcal{R} = (2.40 \text{ kWh}) \left( \frac{\$0.12}{\text{kWh}} \right) \\ &= \$0.29 = \boxed{29 \text{ cents.}}\end{aligned}$$

**Solution (b):**

Using Eq. (III-15), we see that the power of the electric oven is

$$\mathcal{P} = I \Delta V = (20.0 \text{ A})(220 \text{ V}) = 4400 \text{ W} = 4.40 \text{ kW} .$$

The energy to run this over for 5.0 hr is

$$E = \mathcal{P} \cdot \Delta t = (4.40 \text{ kW})(5.0 \text{ hr}) = 22 \text{ kWh} .$$

Using this with our cost-rate, we get

$$\begin{aligned}\text{cost} &= E \cdot \mathcal{R} = (22 \text{ kWh}) \left( \frac{\$0.12}{\text{kWh}} \right) \\ &= \boxed{\$2.64 .}\end{aligned}$$

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