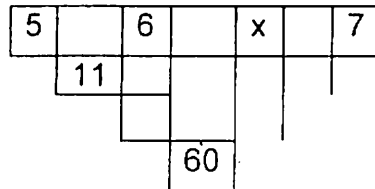


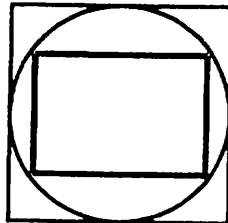
**FRANKLIN MATH BOWL**  
**6<sup>TH</sup> Grade Problem Solving Questions**  
**2006**

*Each team must have ONE complete write-up for each problem. Explain your reasoning. The problems will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.*

1. The odd natural numbers are the numbers 1, 3, 5, 7, 9, ... Find the sum of the first 110 odd natural numbers and explain how you arrived at this answer.
2. Below is an array of squares which contain numbers. The sum of any two consecutive numbers in a row gives the number in the row below and between the two numbers. For example  $5 + 6 = 11$ . Find the value of  $x$ .



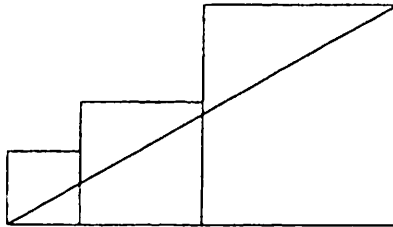
3. A circle has a square inscribed in it and a square circumscribed about it as pictured below. What is the ratio of the length of the side of the larger square to the length of the side of the smaller square?



**FRANKLIN MATH BOWL**  
**7<sup>TH</sup> Grade Problem Solving Questions**  
**2006**

*Each team must have ONE complete write-up for each problem. Explain your reasoning. The problems will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.*

1. A triangle has two sides of length 5 and 12. The third side can be any length greater than 7 and less than 17. What length produces a triangle of largest area?
2. The diagram given below is made up of squares placed side by side. The smallest square has edge length of 2 inches; the second square has edge length of 3 inches and the largest square has edge length of 5 inches. Find the area of the trapezoid in the second square below the line.

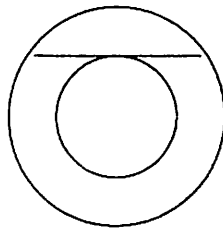


3. Maria is given a rectangular piece of cardboard which is 10 inches by 12 inches. Her mother tells her to roll the paper into a cylinder and stand it on a plate. When Maria completes this task her mother will fill the cylinder with popcorn. Maria can roll the rectangle into a cylinder which is 10 inches tall or into a cylinder which is 12 inches tall. Which one holds the most popcorn and what is the ratio of the larger volume to the smaller volume?

**FRANKLIN MATH BOWL**  
**8<sup>TH</sup> Grade Problem Solving Questions**  
**2006**

*Each team must have ONE complete write-up for each problem. Explain your reasoning. The problems will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.*

1. Consider the sum of fractions  $\frac{1}{1*2} + \frac{1}{2*3} + \frac{1}{3*4} + \frac{1}{4*5} + \dots$ . The symbol  $\dots$  is called an ellipsis and it means that the process continues forever.
- A. What is the sum of the first 8 fractions?
  - B. What is the sum of the first 200 fractions?
2. Two concentric circles are shown below. A chord is drawn tangent to the smaller circle. The length of the chord is the same as the radius of the larger circle. What percent of the area in the larger circle is also contained in the smaller circle? The circles are shown below.



3. Mark is taking a trip and his dad gives him \$510 for his expenses. His money is all in 20 dollar bills and 50 dollar bills. List all possible combinations that he could have.

**FRANKLIN MATH BOWL**  
**Algebra Problem Solving Questions**  
**2006**

*Each team must have ONE complete write-up for each problem. Explain your reasoning. The problems will be graded on your approach, your accuracy, and your communication. Credit will not be given for answers only.*

1. The points (5,6) and (13,10) are opposite vertices of a square. Find the other two vertices. Show how you know that the vertices you chose form a square.
2. Consider the equation  $\frac{xy}{x+4} = 4$ . Find all integer values of  $x$  and  $y$  which make this equation true and write the answers as ordered pairs in the form of  $(x, y)$ .
3. If  $x + \frac{1}{x} = 4$ , what is the value of  $x^3 + \frac{1}{x^3}$  ?

# FRANLIN MATH BOWL 2006

## Group Work Solutions 6<sup>th</sup> grade

1. Students will note that the sums are 1, 4, 9, 16, ... That is to say  $1^2, 2^2, 3^2, 4^2, \dots$ . A pattern emerges, namely the sum of the first  $n$  odd natural numbers is  $n^2$ . So the sum of the first 110 odd natural numbers is  $110^2$ , i.e. the answer to the problem is 12,100.

Alternative: some students might know that  $1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$ . Then

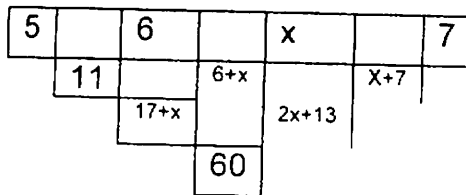
$$1 + 2 + 3 + \dots + 220 = \frac{220(221)}{2} = 110(221). \text{ Thus}$$

$$1 + 3 + 5 + \dots + 119 + 2(1 + 2 + \dots + 110) = 110(221).$$

Therefore

$$1 + 3 + 5 + \dots + 119 = 110(221) - 2 \frac{110(111)}{2} = 110(221) - 110(111) = 110^2 = 12100$$

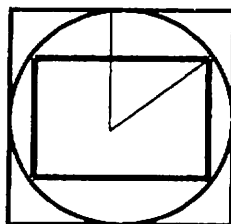
2.



So 60 is  $3x+30$ . Therefore  $x = 10$ .

3. The drawing below is not to any scale, but each line drawn in is a radius of the circle. This gives a larger square with sides of length of  $2r$  and a smaller square of a diagonal of  $2r$  and sides of length the square root of 2 times  $r$

Therefore the ratio is  $2 : \sqrt{2}$ .



**FRANKLIN MATH BOWL 2006**  
**Group Work Solutions 7<sup>th</sup> grade**

1. Let the base of the triangle be 12. If the side of length 5 forms a right triangle, the area of the triangle will be 30 square units. If the angle is not a right angle the height of the triangle will be less than 5, so the area will be less than 30 square units. If the triangle is a right triangle, then by the Pythagorean Theorem the hypotenuse will be 13 units long.
  
2. The horizontal distance is 10 inches and the height is 5 inches, so the line has a slope of  $\frac{1}{2}$ . This means that the second trapezoid has heights of 1 inch and  $\frac{5}{2}$  inches. Its area is  $2\frac{1}{4}$  square inches.
  
3. One cylinder has a circumference of 10 inches and a height of 12 inches. The radius would be  $\frac{5}{\pi}$ , so the volume would be  $\frac{300}{\pi}$ . The shorter cylinder has a circumference of 12 inches, so the radius is  $\frac{6}{\pi}$  and the volume would be  $\frac{360}{\pi}$ . The ratio of the larger volume to the smaller volume is 360:300 or 6:5.

**FRANKLIN MATH BOWL 2006**  
**Group Work Solutions 8<sup>th</sup> grade**

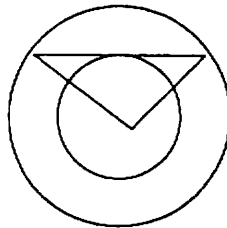
1. The sum of the first 2 fractions is  $\frac{2}{3}$ ; the sum of the first 3 fractions is  $\frac{3}{4}$ ; the sum of the first 4 fractions is  $\frac{4}{5}$ , so the sum of 8 fractions is  $\frac{8}{9}$  and the sum of 200 fractions is  $\frac{200}{201}$

Alternative: note that  $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$ . Thus

$$s_n = \frac{1}{1 \cdot 2} + \dots + \frac{1}{n(n+1)} = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{n} - \frac{1}{n+1} = 1 - \frac{1}{n+1}$$

Consequently  $s_{200} = 1 - \frac{1}{201} = \frac{200}{201}$

2. Draw in the two radii of the circle which intersect



This forms an equilateral triangle. The height of the triangle is  $\frac{\sqrt{3}r}{2}$  by the Pythagorean Theorem. So the radius of the small circle is  $\frac{\sqrt{3}r}{2}$  and its area is  $\frac{3\pi r^2}{4}$ , so the area inside the smaller circle is three-fourths of the total area of the larger circle.

3. Two 50's is the same as five 20's. All numbers must be positive, so find one solution and work from there. Answers (1,23), (3,18), (5,13), (7,8), (9,3)

**FRANKLIN MATH BOWL 2006**  
**Group Work Solutions Algebra**

1. The midpoint of the square is (9,8). The slope of the given diagonal is  $\frac{1}{2}$ , so the slope of the other diagonal (perpendicular) is -2. To move from the center to the point (13,10) one must move right 4 and up 2, so to move to the other vertex one can move left 2 and up 4 to the point (7,12). Similarly the other vertex can be found to be (11,4). The slope of the line between the new vertices is -2. Each diagonal is  $\sqrt{80}$ . There are many other ways to solve this problem, but I like it because it requires the student to understand the meaning of slope and not just use a memorized formula.

2.  $\frac{xy}{x+y} = 4$  and  $x + y \neq 0$ .

$$xy = 4(x+y) = 4x + 4y$$

$$xy - 4x - 4y = 0$$

$$xy - 4x - 4y + 16 = 16$$

$$(x-4)(y-4) = 16$$

The integer divisors of 16 are 1, 2, 4, 8, 16, -1, -2, -4, -8, and -16

This gives answers of (5,20), (6,12), (8,8), (12,6), (20,5), (3,-12), (2,-4), (-4,2), (-12,3)

3. Cube the expression  $x + \frac{1}{x}$ . This gives  $x^3 + \frac{1}{x^3} + 3(x + \frac{1}{x})$  which is 64. So  $3(x + \frac{1}{x}) = 12$ . Therefore

$$x^3 + \frac{1}{x^3} = 52.$$