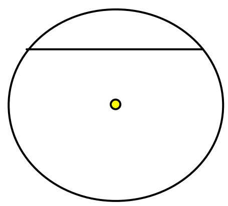
Franklin Math Bowl 2010
Group Problem Solving Test – Grade 6

1. Carrie lives 10 miles from work. She leaves in the morning before traffic is heavy and averages 30 miles per hour. When she goes home at the end of the day, traffic is heavy and it takes her 40 minutes to make the trip home. What is her average speed in miles per hour for the entire round trip?



2. Mary drew the circle above, marked the center and drew in the chord (*the line segment inside the circle*). John measured the length of the chord and found that it was 10 inches long and 4 inches from the center of the circle. What is the area of the circle?

3. Charlie stopped at the gasoline station when his gasoline gauge showed that the tank was one-eighth full. He added 15 gallons of gasoline and his gauge showed that his tank was three-fourths full. If his gasoline gauge was accurate, how much gasoline does his tank hold?

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Group Problem Solving Test – Grade 6 Answers

1. Carrie's round trip is 20 miles. It takes her 20 minutes to go to work and 40 minutes to return from work for a total of one hour commuting time. Consequently, her average speed is 20 miles per hour.

2. By the Pythagorean theorem, the radius of the circle is $\sqrt{41}$. Consequently the area of the square is 41π, or 128.805 square inches.

3. $\frac{3}{4} - \frac{1}{8} = \frac{5}{8}$. Fifteen gallons is five-eighths of the tank. The tank holds 24 gallons.

Franklin Math Bowl 2010
Group Problem Solving Test – Grade 7

1. A rectangular picture has a mat around it that is 1 inch wide. The area of the mat alone is 100 square inches. What is the perimeter of the outer edge of the mat?

2. A man invested some money at 6% simple interest and a different amount of money at 8% simple interest. His interest from each account was the same. What is the ratio of the amount of money invested at each rate?

3. Consider a square with area of 36 square inches. Divide the square into 4 congruent squares by drawing one vertical line segment and one horizontal line segment. If a circle inside the original square passes through the centers of each of these smaller squares, what is the area of the circle?

Franklin Math Bowl 2010
Group Problem Solving Test – Grade 7 Answers

1. This problem can be solved by trial and error, but there is a simple algebraic solution. If the outer edges of the mat are x and y, then its area is given by $xy - (x - 2)(y - 2) = 100$. This simplifies to $2x + 2y = 104$, so the perimeter is 104.

2. $.08x = .06y$.. Therefore $x = \frac{3}{4}y$.. He invested ¾ as much money at 8% as he invested at 6%.

3. If the area of the original square is 36 square inches, then it has sides of length 6 inches. The 4 congruent squares have sides of length 3 inches. The center of each small square is 1.5 inches horizontally form the center of the original square and 1.5 inches vertically from the center of the original square,. The circle has a radius of 1.5 times the square root of 2 inches. The area of the circle is $\pi\left[1.5\sqrt{2}\right]^2$. This is 14.137166 SQUARE INCHES.

Franklin Math Bowl 2010
Group Problem Solving Test – Grade 8

1. A linear function *f* is one that can be written in the form $f(x) = ax + b$ where a and b are real numbers. An example is $f(x) = 3x + 4$. Find the values a and b so that

$f(1) = 3$ and

$f(x) = f(x + 1) – 2$

for all values of x.

2. Every positive integer can be written as sums of powers of 2. For example, $2^3 + 2^4 + 2^6 = 88$. Write 1,497 as a sum of powers of 2.

3. Two lines intersect at the point (40, 30). One line has a slope of 6 and the other has a slope of 2. These lines both intersect the x-axis. How far apart are the x-intercepts?

Franklin Math Bowl 2010
Group Problem Solving Test – Grade 8 Solutions

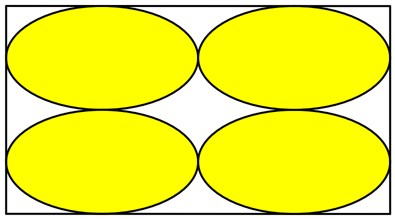
1. This can be done by trial and error or one can use specific values of x. Use $x = 0$; $x = 1$. $f(0) = f(1) - 2$ so $f(0) = 1$. The slope is 2 and the y-intercept is 1, so $y = 2x + 1$.

2. Find the largest power of 2 which is less than 1497. $2^{10} = 1024$; this leaves 473. $2^8 = 256$; this leaves 217. $2^7 = 128$; this leaves 89. $2^6 = 64$; this leaves 25. $2^4 = 16$; this leaves 9. $2^3 + 2^0 = 9$. So the answer is $\underline{2^{10} + 2^8 + 2^7 + 2^6 + 2^4 + 2^3 + 2^0 = 1497.}$

3. The line with a slope of 2 would have a point on the x-axis 15 units to the left of 30 and the line with slope of 6 would have a point on the x-axis 5 units to the left of 30. Therefore, the x-intercepts are 10 units apart.

Franklin Math Bowl 2010
Group Problem Solving Test – Algebra

1. A square has two vertices on the *x*- axis and two vertices on the graph of $y = 1 - x^2$. Find the coordinates of all vertices of the square.



2. The figure above shows four congruent ellipses inside a rectangle with the edges of the rectangle tangent to the ellipses. Identify the lines <u>through the center of the ellipse</u> to the opposite edges of the ellipses as "leps". The area of an ellipse is found by multiplying π by product of the length of the longest lep and the length of the shortest lep and dividing by 4. What percentage of the diagram above is not shaded? Explain your reasoning.

3. Find all values of x such that $\frac{2x^2}{53} + 3x + 53$ is positive.

Franklin Math Bowl 2010
Group Problem Solving Test – Algebra Solutions

1. The width of the square is $2x$ and the height of the square is $1-x^2$ This means that $1 – x^2 = 2x$.

Solve by the quadratic formula and choose the value less than 1. That is $-1+\sqrt{2}$ or .414. The coordinates are (-.414, .828) (-.414, 0), (.414,0) and (.414, .828).

2. Let the longest lep in the ellipse be *a* and the shortest lep have length *b*.. The area of the rectangle is 4*ab*. Each ellipse has an area of $\frac{\pi ab}{4}$ for a total area of πab. The part not shaded has area of $(4–\pi)ab$. The part not shaded is $\frac{4-\pi}{4}$ or 21.46%.

3. Multiply by 53 to get the expression $2x^2 + 3(53)x + 53^2$.. Set this expression equal to zero and solve the quadratic equation. This gives $(2x + 53)(x + 53) = 0$. which has zeros of –53 and –53/2. The expression will be positive for all numbers less than –53 or greater than -53/2.