Math Trivia

The Evolution of Teaching Math

Teaching Math in the 1960s:
"A logger cuts and sells a truckload of lumber for $100. His cost of production is four-fifths of that amount. What is his profit?"

Teaching Math in the 1970s (the new math):
"A logger exchanges a set (L) of lumber for a set (M) of money. The cardinality of Set M is 100. The set C of production costs contains 20 fewer points. What is the cardinality of Set P of profits?"

Teaching Math in the 1980s ("dumbed-down" version):
"A logger cuts and sells a truckload of lumber for $100. Her cost is $80, her profit is $20. Find and circle the number 20."

Teaching Math in the 1990s:
"An unenlightened logger cuts down a beautiful stand of 100 trees in order to make a $20 profit. Write an essay explaining how you feel about this as a way to make money. Topic for discussion: How did the forest birds and squirrels feel?"

Teaching Math in 2000:
A logger sells a truckload of lumber for $100. His cost of production is $120. How does an Enron Accountant determine that his profit margin is $275?"
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- 2-Digit Addition and Subtraction Using a Hundreds Chart
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- What’s Your Credit Score?
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NCTM CONFERENCES AND MEETINGS

Annual Meetings

- 2014
  New Orleans, LA. April 9 - 12

- 2015
  Boston, MA. April 15 – 18

Regional Conferences

- 2013
  Baltimore, MD • Oct 16-18
  Las Vegas, NV • Oct 23-25
  Louisville, KY • Nov 6-8

- 2014
  Indianapolis, IN • Oct 29-31
  Richmond, VA • Nov 12-14
  Houston, TX • Nov 19-21

- 2015
  Atlantic City, NJ • Oct 21-23
  Minneapolis, MN • Nov 11-13
  Nashville, TN • Nov 18-20

Families + Teachers = Mathematical Success

By Missy Carter-Hyatt

In the fall of every year a wonderful change takes place in our communities...school begins. Families and teachers have an air of excitement on their faces and quickly begin making their preparations for the coming school year. One hope these two groups have in common is their desire to receive the best of each other. Families desire a great teacher. Teachers desire a great educational ally at home. These two hopes can only be accomplished through a partnership. Family and teacher partnerships can be formed by understanding the advantages of working together, being knowledgeable of the standards that direct students’ learning, and sharing the classroom with students and their families.

Many studies have shown that across racial and ethnic groups and across school levels, students performed better in mathematics when parents and children participated in discussions about school and the parents were active volunteers at school (Epstein 2009). As educators we have a great platform to foster a partnership with families that the benefits will remain for years to come. “It takes a village to raise a child,” is a quote that was made popular by Secretary of State Clinton in the late nineties and is
still pertinent today. Each child has been influenced and instructed by many adults and peers by the time they arrive in your classroom. These interactions, experiences, and teachable moments all come together to guide our students into adulthood. We have the privilege of being a resident in our students’ villages for only a short time, make it count.

Over the last several months educators across Tennessee have attended meetings to clarify our new Common Core Standards. We need to in turn educate our families of the changes, particularly in math. The majority of parents I come in contact with view math as they were taught: traditional algorithms, rote memorization of facts and procedures, and pages of abstract problems to calculate. It is our job to help families transition into the “new math”. I have found once parents understand why we draw diagrams, break numbers apart, use tools such as the 100’s Chart, number line, and numerous manipulatives they are amazed at their child’s knowledge of numbers. They are certainly more eager to be an educational ally at home. The partnership begins.

The next step in the partnership is to welcome families into your classroom and to help them welcome math into their homes. Request family volunteers during your math block. Every parent loves to see their child in the spot light. Hold a classroom math night where students can volunteer to model their strategies to our classroom families. Assign your students a family homework assignment. Students simply share a problem they solved during class and teach their new math strategy to a family member. The best way to master a strategy is to teach it to someone else. Include current unit math strategies on your webpage and newsletters. The possibilities are endless. The key is continuous communication and a welcoming environment. The partnership grows.

Being an educator and a mother of three I have seen the growth children make when everyone is working toward a common goal. Mutual respect, hard work by both parties, and a love for children can provide “value added” no one can definitively score.
Think, Plan, Teach
By Philip Fields

One of my favorite channels on cable television is Home and Garden Television, or HGTV. I love HGTV because I love to see how other people live and play. I also love to criticize other peoples’ choice on paint, floor covering, or style of house. However, no matter how insidious their paint choice, country where they are buying their vacation home, or size of the backyard one key point always comes out. The one key point that matters more than anything is location, location, location! This key principle holds true in teaching too. This principle was made poignantly clear to me this year.

One of my students’ favorite math activities this past year was my supermarket, oh sorry, the grocery store game. This should have been my first clue. When I said we were going to play the supermarket game, some of the students’ hands shot up quicker than if I had ask who wanted to go to the playground today. The question all had was what in the world a supermarket was. When I told them they laughed and told me in no uncertain terms that it is a grocery store not a supermarket.

What is the supermarket, or grocery store game? The “grocery store” game is an activity I like to use when we discuss rounding and estimating. At the beginning of the school year, I sent a letter home to all the parents asking if they would save old box containers (crackers, macaroni, cereal, and whatever else they bought) and aluminum cans (make sure they are washed out. I did not think I would have to ask that, however I am including that in next year’s letter). If you do not get a large variety of box or cans you may have to improvise a little like I did this year. For instance, I included things on the index cards my family occasionally buys and things I am familiar with from shopping. Here is how the game is played. I take the boxes, cans, and index cards (substituting for boxes...
or cans) and price them just like they were on a shelf in the grocery store. I use yard sale stickers for this task. I always make sure the price is never a whole number since we are working on rounding and estimating. I place the grocery store items around the room on tables and desks or whatever else I can get my hands on. Since I have a small classroom I let the students shop individually; you could easily pair the students for this activity. I give each student $100 in their classroom play money account. I inform them that since they have moved out into an apartment on their own they must go grocery shopping. The one thing they MUST remember is no matter what they buy the check they write at the checkout counter cannot be more than $100 since that is all the money in their classroom bank account. The students are then allowed to go shopping. I also tell the students that the 4th Grade Grocery Store does not allow shoppers to use a calculator; only paper, pencil, and brain. When the students finish shopping they bring their basket to the checkout counter just like a supermarket, oh grocery store. When I have totaled all sales I announce the winner of the activity. The winner this year bought items totaling $98.45. I need to take that student shopping with me next time.

Here is where my lesson started. I had help making the index cards. My son helped me make the cards and price them. While I was totaling the students’ grocery items I noticed a trend in their purchases; what they did buy and what they did not buy. Each one of the students bought home-made sausage, soda, gravy mix, biscuits, bacon, coffee, flour, corn meal, Crisco®, candy, cake mix, potatoes, and hamburger helper. What surprised me was what the students did not buy. None of the students bought vegetables, Kool-Aid®, milk, fruit, rice, noodles, and I-Tunes® gift cards. No one bought an I-Tunes® gift card? My son could not survive without his I-Pod® when we travel. When I asked them about this they told me no one has internet service. Only one student in the class has internet service. When I told my son this he could not believe what I was saying. His question to me was, “How do they survive without X-box® live?”

This activity really got me to thinking. What else was I doing or saying that was not in the contextual realm of my students’ world? After this lesson, I really tried my best to step back and look at the topic I was going to present to my students. By doing this I did found out there were times I had to spend 10, 20, or even 30 minutes setting the stage for our next topic of discussion. Another thing this activity has forced me to do is make another sign for my supermarket. It now reads 4th Grade Grocery Store.
The Alpha Website to Use in Math Classrooms!

By Melissa Trinkle

Today, like most days in Dr. Nivens’ Mathlete’s classroom, I was introduced to a math tool that made me wonder how I have taught without it for so long. Today’s math tool was wolframalpha.com. As Dr. Nivens was showing our class this website I know I was staring in amazement. It is quite possibly one of the coolest websites ever! The ideas to use this website in a math classroom are limitless. I immediately thought, “why has no one showed this to me before and how am I going to use this in my 7th grade math classroom”. If you are also behind on this technology, you are in for a treat!

WOLFRAM|ALPHA is an engine for computing answers and providing knowledge. Do not get it confused with a search engine. WOLFRAM|ALPHA uses built-in knowledge curated by human experts to compute on the fly a specific answer and analysis for every query. You can find this information and more by taking the “tour” at wolframalpha.com. Basically, you type in what you want to calculate or know more about, and in seconds, the information is right in front of you. Disclaimer: the site will solve math problems! Teachers might want to keep that aspect to yourself until your students figure it out for themselves. You know they will!

It is somewhat overwhelming thinking of all the ways wolframalpha.com could potentially be used with 7th graders. I could incorporate this website with every standard I teach but it might begin to lose it’s “coolness” if used too often. Below are examples of how I would use the website in my own classroom.

Anyone who has ever taught in a middle school knows that middle school students are all about themselves. I think my 7th graders would love a project like this. If you go to wolframalpha.com and type in your name it will give you oodles of information about your name. One activity that could be used goes along with how many “Fill in your name” are born each year. I would have students solve using proportions how many of “their names” are born each month, 4.5 months, 10 months, daily, weekly, every minute, etc. You students can discover whose name is most common and whose names are unique. Imagine all the possibilities!
(Computation and Estimation: SOL 7.4-The student will solve single-step and multistep practical problems, using proportional reasoning.)

Staying on the topic of yourself, if you scroll down the screen you will find different graphs. Each student in your classroom will have meaningful data to use in constructing their own histogram. One graph shows an estimation percent of age distribution. For example, with my name, the average percent between the ages of 0-9 is 0.5% of all Melissa’s in the United States.
The highest is ages 30-39 with an average close to 4% of Melissa’s in the United States. Instead of percents, I would have them find the actual population. If you aren’t sure how many Melissa’s are in the United States, guess where you can find the information? wolframalpha.com! Not sure what 4% of 719311 is, you guessed it? Wolframalpha.com!

(Statistics: SOL 7.11 a-The student, given data in a practical situation, will construct and analyze histograms.)
Do I Have to Show My Work?
By Deborah Smith

This year, for the first time, my colleagues and I observed a significant majority of our students showing their work while taking multiple-choice tests. We witnessed students going back over a test, reworking difficult problems, and checking their work. We noted students doing the easier questions first and return to the more difficult questions later. We even noticed students circling key works, marking out distracting information, and using the process of elimination by marking out answer choices.

In previous years, I had always expected my students to show their work, but few complied willingly. What caused the difference? Why did my students suddenly start using these strategies while test taking? The Vance seventh grade math team collaborated and created a war plan for teaching our students how to take a multiple-choice test, such as a benchmark that the students take every quarter. We wanted to use a basketball theme, and the design resulted in a playbook of sorts. As a team, we matched various test-taking strategies with seventh grade math standards. Students would record the test-taking tip on the left page of their playbook, and use that strategy to complete a multiple-choice assignment on the right page of their playbook.

Nevertheless, in my opinion, the real magic occurred when we incorporated a song, video, or picture book to make the strategy fun and memorable for the students. The seventh grade math team coordinated a math skill, with a test-taking tip and a song. One of my favorites is the U2 song, “Stuck in a Moment”. The chorus pairs well with the strategy of pacing during a test, encouraging students to not get stuck on one problem and losing valuable time. I enjoyed playing the song for the class to see if the students could figure out what the test-taking tip of the day was. After the students discovered the “play of the day”, I would model how to use that strategy before giving the students an assignment for them to practice. The classroom communities improved as students began sharing strate-
gies with each other that they had successfully used then pass those suggestions on to me. After the students discovered the “play of the day”, I would model how to use that strategy before giving the students an assignment for them to practice. The classroom communities improved as students began sharing strategies with each other that they had successfully used then pass those suggestions on to me.

At the conclusion of this unit, my students wrote a reflection piece. Many students stated that having these test-taking strategies in their pocket gave them a boost in their confidence when they sat down to take a test, in not only my class, but also all their academic classes. The students felt that their anxiety decreased because they knew the material and possessed tactics for taking the multiple-choice test. Teachers from other academic areas also commented that the difference was also remarkable in their class.

I have included the results of the collaboration efforts of Vance’s seventh grade math team. You are welcome to use what you see here or adapt it to your needs. All of the songs were available on You Tube when this plan was initially developed. A word of caution, however; when surfing around the web, be sure to listen to each song and view the video in its entirety as if you were a student. Some lyrics are obvious in their unsuitability for use in the classroom, but still other songs may have more subtle lyrics that students may misconstrue.

Please feel free to contact me. I would be happy to answer questions or hear about your success!

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<tr>
<th>Lesson #</th>
<th>Coach Lesson</th>
<th>Test-Taking Strategies</th>
<th>Song</th>
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<td>Pick out the easy questions</td>
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<td>3</td>
<td>Comparing Rational Numbers</td>
<td>Don’t change your answer unless you are sure you need to change it</td>
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<td>Relax, don’t do it. Gonna Rock This Test –</td>
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<td>Changes in Area and Volume</td>
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Reading Math
By Stephanie Miller

Extra! Extra! READ all about it! In most people’s minds, Reading and Math are two completely different subjects. However, literature can be used in the Math classroom for several purposes. One reason is to reach those students who are naturally better at literacy skills versus math skills. Another reason is to teach students that Math is just a language that must be translated. This in turn can benefit students who are required to take constructed response tests. These are just a few examples of how including literacy skills in a math classroom can maximize student learning.

“Best practices” tells us that integrating various subjects sets a better foundation for comprehension. Not only can teachers connect math to literature, but also to science, social studies, and fine arts. I have found that students, including myself, are typically either Math/Science oriented, Literacy/Social Studies oriented, or artistically oriented. Just to prove my point, I will use my siblings: I see things mathematically or scientifically, my sister is talented linguistically and can pick up any language she hears, and my brother is the artist in the family when it comes to any imaginable instrument. So for the student who is more comfortable with literacy than abstract math skills, using literature with math content puts the concepts into a familiar media that the student can be more successful with. I also recommend incorporating art, music, movement, or anything else that makes the math meaningful to each and every student.

Math can be expressed in many languages – words, pictures, symbols, and numbers. Students need to understand that numbers and symbols are just a mathematician’s abbreviation of the written or verbal words. Exposing students to math literature helps them grasp this connection. Any math problem, even a simple computation, can
be expressed using words. For example, \(2 + 2 = 4\) might mean, “two points plus two points equals four points.” As students become fluent in the language of math, they will be able to see words and express it with pictures, numbers, and symbols and vice versa.

The ultimate goal is to get students to develop fluency in translating between each form of math. The Constructed Response Assessments (CRAs) that Tennessee is moving towards almost demand that our students be able to do this. We are no longer interested in, “Can they reach the correct answer?” but also, “Can they explain how they got there?” This requires an ability to verbalize the justification of the answer, even if the answer itself is wrong. Students, and teachers, will need practice doing this in the classroom if they are expected to accomplish on a test.

Math literature is an excellent way to model the process of putting math concepts into words. It has been said that the best way to know if you understand something is to explain it to someone else. That is what we are asking of our students, and it is our job to model this for them. We need to differentiate our instruction to meet every child’s needs, teach them the language of math, and train them so they know what to expect on constructed response tests. Reading math is the optimal way to accomplish all those goals at once.

**HOW DOES IT KNOW?**

A physicist, a mathematician, and a mystic were asked to name the greatest invention of all time. The physicist chose the fire, which gave humanity the power over matter. The mathematician chose the alphabet, which gave humanity power over symbols. The mystic chose the thermos bottle.

"Why a thermos bottle?" the others asked.

"Because the thermos keeps hot liquids hot in winter and cold liquids cold in summer."

"Yes - so what?"

"Think about it." said the mystic reverently. That little bottle - how does it know?"

http://www.onlinemathlearning.com/math-trivia.html
Like thousands of others, I was deeply saddened by the recent passing of actor Andy Griffith. Although I admired the wide range of talent he displayed in various movie roles and as attorney Ben Matlock, I have to admit for me Andy Griffith is and always will be synonymous with Sherriff Andy Taylor of Mayberry, North Carolina. Growing up I visited with the Taylors every Monday night, and as a grown-up, I revisit Mayberry whenever TV Land gives me the opportunity.

I seriously doubt that an episode of The Andy Griffith Show exists that I have not watched—numerous times. Just in case you are not a member of TAGSRWC (The Andy Griffith Show Rerun Watchers Club), here is a brief synopsis. Sheriff Andy Taylor is a widower with a young son Opie. Andy’s Aunt Bee keeps house for the two of them. The show depicts small-town life by allowing us to see the interactions of the Taylors with friends and neighbors like Deputy Barney Fife, barber Floyd Lawson, and filling station employees Gomer and Goober Pyle. Some of the show’s most popular installments depicted auditioning for a solo part in the town choir, the buying of a first car, and canning homemade pickles. However, it is an episode entitled “Opie Flunks Arithmetic” that can teach us math teachers a few lessons.

Now Opie is a well-mannered, intelligent little boy who usually does well in school. One day Miss Helen Crump, Opie’s teacher, stops by the Mayberry Courthouse to let Andy know that Opie has a D in arithmetic. She explains that sometimes a little extra attention is all that is necessary to correct such problems. Andy agrees and tells Miss Crump that he will help Opie with his math. As Miss Crump is leaving the courthouse, Opie stops by. Andy asks his son what seems to be the trouble with arithmetic, and Opie explains that the problem is long division. He tells his “paw” that he makes so many mistakes and erases so much that he tears holes in his paper. Opie is convinced that he could make a better grade if he just had better paper!

Later that night Andy sits down to help Opie with his math homework. The first problem is 169 ÷14. The following is a transcript of the conversation between Andy and Opie taken from a blog by Holly McLeod of Landrum Human Resources Company:

Andy: Put your 14 down, draw a line up and over and put your 169 down there.
Opie: Why do you do that, Paw?
Andy: ‘Cause that’s the way you do it.
Opie: When we give an answer we have to tell why we do something.
Andy: The reason you put the 14 down on the left like that is, is, uh, uh, is because, uh… that’s the rule.
Opie: She’ll ask us why you put it on the left instead of the right.
Andy: She will? Well, uh… we probably better not tackle too much the first night. Now how many times does 14 go into 16?
Opie: Once.
Andy: Put your one on top of there. Ok, now what you got left over?
Opie: Two leftover.
Andy: Bring down the nine. Now, how many times does 14 go into 29?
Opie: Twice.
Andy: Twice, that’s it. Now what you got left over?
Opie: One left over.
Andy: That’s a time! Now, put down a decimal point and two zeroes.
Opie: Why?
Andy: What do you mean, why?
Opie: Just why?
Andy: Well, uh… you put down your decimal point because you’re gonna put down two zeroes after that. (Opie just looks at him.) The reason you put down a decimal point and two zeroes is so’s that you can, uh, keep dividing, see, and shove that number on out there, and that way you can keep thinking about it and working on it and everything, until you finally get it done. You understand?
Opie: No.
Andy: It’ll come to you. (McLeod, 2011)

Filmed in 1965, “Opie Flunks Arithmetic” portrays the way most children of that generation were taught “long division,” including myself. Obviously, the procedures involved are hard for Andy to explain. This is probably because Andy memorized the steps of the algorithm (divide, multiply, subtract, bring down) but attached no meaning to the different steps. It is hard to explain something you do not understand. Watching this episode, I can appreciate both Opie’s struggles as a student and Andy’s frustrations as a parent.

So how would we approach the teaching of long division today? Using the pedagogy of Dr. George Poole, Professor of Mathematics at East Tennessee State University:
Opie should have been allowed to construct his own meaning of division by using counters, cubes, and other manipulatives to depict various real-life problems. These problems needed to be of both a partitive (group) nature and a quotative (size) nature. Opie also should have had the opportunity to work with inexact quotients. (Poole, pp. 146-152)

Aunt Bee leaves a jar of her homemade pickles at the courthouse for Andy and Barney. There are 11 pickles in the jar. How many pickles could Andy and Barney eat if they share equally? Would there be any left? (The quotient is 5 ½.)

Opie’s friend Johnny Paul Jason has 37 baseball cards. He buys an album from Walker’s Drug Store to store his cards. Each page holds 5 cards. How many pages will Johnny Paul’s album need to store all his cards? (The quotient is 7 remainder 2, but the album needs at least 8 pages to accommodate Johnny Paul’s cards.)

Once Opie demonstrated understanding of the division principle, Miss Crump should have provided opportunities for him to gain confidence and speed with problems that have exact quotients (the division facts). She then needed to introduce the idea of power ten quotients: dividing larger numbers by using knowledge of the division facts and then subtracting the number of zeroes in the divisor from the number of zeroes in the dividend.

Miss Clara Edwards has won 1200 ribbons for her homemade pickles. She has entered pickle contests for 20 years. How many ribbons does she average winning each year?

1200 ÷ 20 = (12 ÷ 2) with 1 zero (60)

Providing Opie a strong foundation in place value would have prepared him for the strategy of breaking apart larger dividends. (Poole, pp. 208-210)

Aunt Bee processes 169 cucumbers. She puts them into 14 canning jars. How many pickles did Aunt Bee put in each jar? One possibility:

(100 + 60 + 9) ÷ 14 à (140 + 20 + 9) ÷ 14 =
10 + [(20 + 9) ÷ 14] =
10 + [(28 + 1) ÷ 14] = 12 with remainder 1

This is the answer to Opie’s homework problem, and I believe Opie, unlike Andy or me, would have been able to explain how he got his answer should Miss Crump ask him.

However, the strategy that I believe would have been Opie’s favorite is one presented by Dr. Poole on July 16, 2012, in his Math 5010 Problem Solving class. This method combines the use of partial quotients and estimation with “bumped-up” divisors and smaller dividends, allowing students such as Opie to “sneak up underneath”

\[
\begin{array}{c}
14 \div 169 \quad 5 \quad 100 \div 20 = 5 \\
\underline{70} \quad 60 \div 20 = 3 \\
99 \\
42 \quad 40 \div 20 = 2 \\
57 \\
-28 \quad 29 \div 14 = 2 \text{ remainder } 1 \\
29 \\
\underline{-28} \quad 1 \text{ (remainder)}
\end{array}
\]

Why do I think this strategy would be Opie’s favorite? He would never have to erase!

Works Cited

Previously I had used a hundreds chart to teach skip counting, greater than and less than, and place value. This year I used a hundreds chart to help my second grade students solve 2-digit addition and subtraction problems. You can count on by tens on a hundreds chart by moving down a row in the same column. You can count on by ones by moving to the right in a row. The same is true for subtraction except you move in the opposite direction. You count by tens as you move up a column to subtract tens and to subtract ones, you move to the left in the row.

Each child in my class had a hundreds chart and after much practice adding tens to 2-digit numbers and adding ones to 2-digit numbers, I asked the students how many rows down they would have to go to add 54 + 18? Start at 54 and go down 1 row to show 1 ten in 18. Ten more than 54 is 64. Then count on 8 more to the right to show the 8 ones. The correct answer is 72. I continued giving the students problems to solve until they were comfortable with this strategy. The students loved this strategy and used it to solve two digit addition problems with and without regrouping.

After several days of solving addition problems using a hundreds chart I asked the class if they thought they could use the hundreds chart to solve subtraction problems. We thought back to when we began adding using a hundreds chart and one child reasoned that on a number line you count on to add and count back to subtract, and on the hundred chart we had learned to count down to add tens and to the right to add ones, then you must count up on the hundred chart to subtract tens and to the left to subtract ones. To solve 67-24, begin on 67 and count up 2 rows in that column, that is 47. Then count to the left 4 spaces to subtract 4 ones and the answer is 43. We tried the strategy and the class discovered that it worked. They were so excited about a way to solve 2-digit addition and subtraction that was fun and did not require using pencil and paper. I was excited to see them being successful with a strategy that made math fun. I also found that they had a greater understand of place value after using this strategy.

I found that even after we went on to other standards in math, some students continued to use a hundreds chart to solve 2-digit addition and subtraction problems. Some of my top students were able to take it a step further and utilize their greater understanding of place value to solve these problems mentally. I will definitely continue to help my students discover this strategy in the future. I feel that if I had been encouraged to use different strategies when I was in elementary school I would have had a better understanding of math and definitely enjoyed it more.

Resources
Why Does Math Take The Backseat?

By Chase Watkins

I hear students say all the time, “I can’t do math,” or “I hate math!” Although, rarely do I hear a student say, “I can’t read.” Why is this? Everyone can pick something they like to read about, but can we as teachers find something in math that interests our students?

In grades K-5, you will find teachers having reading lessons first thing in the morning; the best and most attentive time for learning. We have words of the week; we break down words, use them in sentences, and look in depth at their definition. When we read our stories, we look at the main idea, characters, and relate them to real life situations. Many of our classrooms have thirty minutes set aside for free reading time. If a student finishes his or her work early, the number one response from a teacher is “read a book.”

Do we have to set a schedule that places reading before math every day? In reading, we analyze each vocabulary word. Could we not have a number of the day where we look in-depth at numbers? Numbers could be broken down, showing their place value with expanded form and manipulatives. We could write the number in word form, possibly have spelling tests where students spell the number words. Students have to be able to read to do problem solving, so can we not integrate these two subjects in order to connect the two content areas for a more beneficial type of learning. We could use story problems instead of just short word problems. We could talk about main idea, characters, and solve math problems found within the story. We could also assign our students a certain number to write a story. For example, students could write everything they know about a certain number, as well as, how they see it and how they relate this number to their life.

Math not only takes the back seat to reading in the elementary grades, but even at the college level. While working on my education degree in college, the requirements for reading doubled the math requirements. During the math methods classes, I did not learn effective teaching strategies in order to teach math. Using math manipulatives was rarely mentioned as an effective learning tool.

Why is math in the back seat? I have often wonder if we spent the same time for math as we do reading, would our students be less apprehensive about learning math and possibly more successful in acquiring math skills? We know it would not solve any problems to move math to the front and reading to the back, but could we not let math ride shotgun while reading is in the driver’s seat?
Bottle Math

By Sherry Strickler

A great versatile math manipulative simply consists of a plastic bottle and lid with dice inside. I like to use plastic juice bottles because they are smaller and easier to store. You can vary the amount of dice inside according to what skill you want to reinforce.

**Addition:** Start with 2 dice inside the bottle. Partners take turns shaking the bottle and adding the numbers inside. They can record their addition sentences and compare who has the greater or lesser sum, odd or even sum, or whichever skill you want to reinforce. For more of a challenge add more dice so the students are able to add 3 numbers, 4 numbers or more. For 2-digit addition use two dice, shake the bottle, point the lid side of the bottle to the left. Partners take turns rolling the dice and recording their numbers, then partners work together to add the addends. This can be changed to any size number.

**Place Value:** Start with 2 dice inside the bottle. Turn the lid to the left side. You could place a sticker on the left hands of the players to help them find the left side. Partners read and record their numbers. Use the greater than and less than symbols. Decide what the winning target will be. (Greater or lesser, odd or even) Circle the number that is the winner and play again. See who won the most out of 10 tries. Add as many dice to cover the place value you are practicing.

**Subtraction, multiplication, and division:** These skills could also be practiced using the dice in the bottle and changed to fit the particular skill.

This is also a nice way to use dice without the pieces being misused (or thrown across the room).
Changing Math Ideas
By Valerie Moore

With the changes in our education system and the switch to the common core standards and constructive response tests in Tennessee, we as teachers must make changes in our own thinking and teaching strategies. We no longer should just be satisfied with simply the right answer on math problems, but rather we must question our students on how and what concepts they used in order to get that answer.

Attention to detail in the written answer is the key to student success on the constructive response test, but ultimately the students will be successful if they look closely at the details in the real world applications when everyday problems arise. As math teachers we should show our flexibility by looking beyond one answer. Students will arrive at correct answers to problems by using different concepts and algorithms, possibly with a different method.

The Eastman Scholar Mathlete Program gives teachers the opportunity to broaden their ideas in teaching math concepts while becoming more flexible in teaching strategies. Mathletes are also learning to pay close attention to details when evaluating answers, as well as the problem. This program allows teachers from the area to become a network that provides new and great ideas for teaching math. Using professors from East Tennessee State University help bridge the gap between what is being taught at the university level to aspiring teachers and what is currently taught in classrooms. Teachers who are involved in this program have learned how to embrace these changes and are fortunate to have Eastman and ETSU as partners in education.
A Mathematical Mindset
By Whitney E. Loggans

I never really thought of myself as a math student throughout elementary, middle, or high school. I was actually terrified of mathematics! It was not until college, at East Tennessee State University, that I really thought - “I can do this!” The mathematical mindset can be very intimidating, or at least I thought so. While obtaining my undergrad for Elementary Education K-6 at ETSU, I was required to take a total of three math classes. My mindset, of course, was that of a college student: hurry up and get finished, so I can get my degree and start working! I learned a lot of neat techniques in math, but I didn’t really appreciate what was being taught to me. I did, however, begin to feel comfortable with math by the time I started student teaching.

I graduated from ETSU in May of 2009, and I did not receive my first teaching job until June of 2011. I worked as an Instructional Assistant for Kingsport City Schools, and had the wonderful opportunity of being offered a third grade teaching position with KCS. When I first started as a new teacher I was very worried about teaching math, because I was always apprehensive about math as a child. In Kingsport City Schools we use the Math Investigations curriculum. Therefore, when I started teaching math I felt more comfortable because all of those great strategies that I had learned in college were coming in handy! I really love the variety of mathematical strategies that are used within the Investigations curriculum.

Despite the fact that I was very nervous about math growing up, it is now my favorite subject to teach. My “this is where it all clicks” point for mathematics was in college and when I actually started teaching the material. Once I realized that mathematics can be done in a variety of ways, my mindset changed significantly. I can honestly state, now, that I really do love math!
The definition that I would use to define positive discipline is “to catch a child doing something good and telling them”. When you hear the word discipline “positive” does not even come to mind. The word discipline come from the word disciple which means to teach, not punish. Unfortunately, the latter is how it is known. “The most effective forms of discipline encourage good conduct by building a mutually respectful bond with the child, letting her know ahead of time how to act, and praising mature behavior” (Zahn-Waxler & Robinson, 1995, pg. 143).

How is a child’s brain affected by positive discipline? Rousseau, a French philosopher, stated “Children are noble savages, naturally endowed with a sense of right and wrong and an innate plan for orderly, healthy growth” (Berk, 2008, pg. 12). Rousseau felt that adults would only harm their inborn traits. He thought children controlled their own destinies. However, in this day and age the environment does not allow children to have this control. “Child development psychologists are discovering more and more about the influence of the environment on children’s brain development, both before birth and during the beginning years of life. The Constructivist belief that children construct knowledge from their interactions with their physical and social world now takes on an even deeper meaning” (Fields & Fields, 2006, pg. 13).

The main goal of using positive discipline is to make sure the children feel that they are in a safe environment, and that they will not be chastised for answering a question wrong or being made fun of in any way. “All people (including children) deserve dignity and respect. A basic principle of Adlerian psychology-- the philosophy of maintaining dignity and respect for all humans” (Nelson, Lot, & Glenn, 2000, pg 24).

“Humiliation and shame are not effective motivators. Over the last decade, a number of professionals concerned with children (such as pediatricians, psychiatrists, and social workers) have formally adopted positions in opposition to spanking or humiliating children in any way. These specialists have paid attention to the research demonstrating that the long-range damage to children far outweighs the immediate advantage of controlling behavior through punishment.” (Nelson, et al., 2000).

Positive Guidance encourages an environment where children make their own decisions and choices. The children
are learning self-discipline, or moral autonomy.

“Moral autonomy means having the ability to make decisions about right and wrong, regardless of any rewards or punishments, yet taking into consideration the rights and needs of all involved” (Fields & Fields, 2006). Each choice is a continuance that develops the brain. If the child is not behaving appropriately, the teacher must find the cause of the action in order to help that child.

Teachers can use positive discipline in the classroom by learning strategies that include the children in the daily class activities. “Teachers need to teach and encourage not be policemen, judges, and juries. To stimulate the children’s brains the teacher can suggest that the children aid in making the classroom rules, set a daily routine, show respect to his / her students and also conduct class meetings where the children can bring issues to be resolved” (Nelson, et al., 2000).

Positive Discipline is a wonderful concept and plan. I agree with Dr. Nelson’s statement about “humiliation and shame”. I myself was brought up in a society where fear and resentment were the disciplines of choice. We were never given choices, only consequences. If the students did not comply with the rules at school, the students knew that something bad was going to happen. If we got a spanking at school, we could expect a spanking at home. I don’t feel that I respected my teachers. I feared them and my parents.

Positive Discipline effects brain development in numerous ways. We are born with billions of neurons. It is these connections that ensure how we survive. It begins at birth and is continued throughout the entire life span. Positive Discipline gives a child a sense of control in his or her personal chaos. These positive connections that are aided by parents and caregivers give the child the ability to make choices. Children have the right to make mistakes and to learn from the consequence, free from punishment. It is important that they feel that children are in a safe environment so they will attempt to make a decision. Children are also in a safe environment where they are free to answer questions without the fear of humiliation or ridicule. By allowing the children these options, they begin to feel respected. If the children feel respected they will show respect. Teachers have many options to rear children. I feel that positive discipline is the best method. It allows children to be in control of their decisions with the teacher there as a guide. The teacher is not there to punish but to encourage. Will all schools be able to practice “positive discipline”? How long will it be before we see the effects of this method? We as teachers need to become encouragers and help students achieve long term goals, and most of all, strive to become lifelong learners.
Practically Perfect Pacing: Is it Possible?
By Carolyn Yvelle Mull

Hopefully, at some point in your life, you have been fortunate enough to be in a classroom that was an absolute joy. On the other hand, all of us have been subjected to hours of classroom torture where each minute painfully passed. I recently attended two math seminars scheduled a few weeks apart. One was only a one-day seminar, and the other seminar was a week long class. Sadly, the one-day seminar was torture. However, the other class was one of those great learning opportunities where the teacher knew how to pace the day (No, it was a week!) for enjoyable learning.

What made the one-day class a day of misery? The one-day program was a pre-packaged program. The instructor of the one-day class did not do any type of pre-assessment at the start of the seminar. She made out her lesson for the day without attempting to know if it was appropriate for the group of learners. Sadly, it was not. Secondly, the environment was not conducive to learning. The group was too large, the room was too big, and the entire lesson was addressed to the whole group rather than breaking off into discussion groups or partners. Finally, the timing of the lesson was off... way off. Of course, the timing issue was not the instructor’s fault. This was a variable that was out of her control. It was the end of the school year. Many of the adult students (teachers) in the class had other pressing issues on their mind. TCAP scores had been released, possible job changes were in the works, and several non-tenured teachers were wanting to know if they even had a job next year. However, even in the face of difficult circumstances, we have all seen awesome teachers strategically overcome at least a good portion of uncontrollable variables by taking charge of the factors that were within their power.

What changes could have been made to overcome the instructor’s obstacles that day? Taking time to get to know the group, pre-assessing prior knowledge, and trying to determine what the learners wanted to learn could have helped her adjust the lesson to make it more engaging. In addition, breaking off into groups and completing activities in small groups could have alleviated many of the environmental factors that were a distraction; thus, creating the opportunity for the teacher to extend learning for the small group divisions. Unfortunately, even when we have...
controlled all possible factors in our power, lessons occasionally nose-dive.

Not one single teacher I have ever met had a goal of trying to create boring lessons or to torment students with meaningless activities paced totally wrong for students. Yet, (I am going out a limb to say this) the majority of teachers still struggle with this at least on some days, during some lessons, or during some “non-teachable” moments. Of course, I do not make this comment as a research-based statement. My thoughts are just based on years of personal experience (as a student and as a teacher) as well as other teachers’ tales of woe.

As educators, we all lament when we feel like students were not engaged, excited, or even remotely interested in the lesson. However, when the stars align and students enthusiastically engage in every learning activity, we, as teachers, are thrilled and jubilantly share the details of the lesson with our teacher friends. These teacher friends then scurry to try the same lesson. After all, we have collaborated, and their students usually have the same areas that need strengthening. Why does the same lesson, then implemented the same way in the other teacher’s classroom not always achieve the same glowing result? The students need the same learning after all, right? They are from the same demographic area, same age, same balance of learners in the room, and so forth.

I will admit this has happened to me more than once. When I hear a teacher rave about how great a lesson went, I think I just have to try it. Many times, fantastic lesson ideas from other teachers will also do amazingly well in my room. However, there are occasions where these same lessons that were gloriously successful with others totally flopped in my room. After a failed lesson, I wonder why does this happen, and why is it that a favorite lesson that has worked wonders year after year suddenly fails the group of students I have now.

This is where teacher math expertise must come into play. First, we need to be able to analyze all the assessment data to see what standards still need to be mastered. That provides one of the important planning components necessary for engaging students. The data will tell us what students are on the ‘cusp’ of learning so that we can engage students without frustrating them. Of course, determining the standards to be taught is only one tiny piece of the puzzle of developing a great lesson.

The vast majority of the problem is considering the seemingly infinite variables that are part of the unique
classroom equation for success. We have to use our math skills to calculate many variables, determine our probability for success, somehow assign values to abstract personality factors, and mentally calculate the interest factor given the age level and demographics of our students. Once our trial and error calculations are complete and the figures predict an outcome of success for our students, we are ready for a perfectly paced lesson.

We know from personal experience that enjoyable classrooms are those where the day flows naturally. The teacher relates to the students in a natural, relaxed manner. Why does this occur? Because teachers with perfect pacing know the data, but more importantly, know what students need, know what students like, know how to group students successfully, know how to use classroom resources in the most efficient and effective manner, know when to move on from each lesson segment, and know how to assess at the end of the lesson to plan for making the next perfectly paced lesson. The lessons are perfectly paced because the teacher knows the students. Fortunately, seasoned teachers also know it is okay if the lesson does not go as planned.

As school starts this year, we should, of course, get to know our students’ data. However, our biggest challenge is to really get to know our students so that we can adjust and be flexible to meet our students’ needs that are ever-changing. The variables will constantly change, but great educators consistently adapt to meet those needs. Know, know, know your students! This just might be the most important variable for success in your unique classroom equation this year!
What’s Your Credit Score?

By Dawna M. Owenby

I have never met anyone who isn’t interested in money….in getting it, keeping it and/or spending it. That is especially true of kids! Many have watched their parents pull out a credit card and run it through the machine at the store, if they don’t have cash on hand, to make a purchase. But do they understand the principle of credit and are they learning to be responsible in paying the bill for that credit card that comes in every month? If you pay your bill in full and on time then you can build your credit score which allows you the opportunity to apply for and possibly receive loans for large purchases should the need arise. However, when you are late in paying your bill or don’t pay that month’s bill in its entirety, there are some serious consequences. One such consequence is having a low credit score rating which could keep you from being able to get a loan from the bank, credit union, or a car company.

Let’s begin building our children’s understanding of this in a meaningful way at school which rewards good choices and is a deterrent to poor choices made by students. In a system created by some colleagues of mine in North Carolina and further developed while I was a team member at the school, each child receives a “credit card” made out of cardstock. It is about the size of a credit card and has space for the child’s name and a table outlining the twenty weekdays in a one month period (see attached sample). The spaces are left blank so that the dates on the card can be customized each month. Students carry these cards around with them in their pencil pouch or in the cover of their binder to each class. When good choices are made by the student the card is left intact. However, as students make poor choices his or her card is punched by the teacher as a consequence using a small-pinned hole-puncher. At the end of each 20 day period the students are given the opportunity to participate in a special activity set up by the teachers. The idea is that the fewer the punches, the more time the student can spend in the special activity. Therefore, the goal is to get no or at least as few “punches” as possible. If a student receives no punches in the 20 day period, the student participates for the entire hour of the activity. If 1-3 punches are received, students may participate after 10 minutes have expired. If 4-8 punches are received, students may participate after 20 minutes out. If 9-13 punches are received, students may participate after a 30 minute time period. Finally, if a student has received 14 or more punches, the student may not participate in the activity at all. One teacher begins the reward activity and the other teachers take turns taking the groups of students in the other categories and bring them to the predetermined location at the appointed times. While the students are waiting to participate, they will use the time to read material of their choosing or make up late work.

Punches are given by teachers for a variety of reasons. Listed below are some of those reasons and their corre-
sponding codes for the various infractions. However, they are not limited to the following:

1. No homework – NHW
2. Back to class (unprepared) – BTC
3. Talking – T
4. Not following directions – NFD
5. Not Paying Attention – NPA
6. No Master Card – NMC
7. Misbehavior – MB
8. Lost Behavior Chart – LC

Not only is a student’s card punched, but the reason for the punch is written by the teacher on the behavior documentation record (see attached). Note that each teacher is assigned a certain pen color with which to document so that not only does the parent know why the card was punched but also what teacher did it. This eliminates any confusion on the parents’ part and also allows the parents to know who they would need to contact if they have any questions.

On Friday of each week, the students take home the Behavior Chart to have it signed by a parent. It must be returned on Monday signed or he/she will be given a punch in the card and one additional punch for every day after Monday that it is still not signed. If a card is lost, a new one will be given and the new card will be punched for having lost the first one. If a second card is lost within the 20 day period, the student automatically loses the privilege of attending the special activity regardless of the number of punches previously given. Therefore, emphasizing to the children the need to be responsible for his/her actions or inactions. At the beginning of each month the students are given a new card and the cycle begins again with a fresh start. Tomorrow is a new day with no mistakes.

Some ideas for reward activities include: watching a movie, kite flying, kickball game, Christmas $1.00 gift exchange, decorating Valentine bags, board games, free time on playground, Bingo with small, donated prizes, dance, and a beach themed party. The ideas are endless, but the benefits are great. Students focus not only on good behavior, but getting his/her work done, turned in and on time at that.
Name_______________________________________

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(Master Card sample)
# Behavior Management Chart

Child's Name _______________ Date __________

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## Teacher Code:
- Blue
- Green
- Yellow
- Red
- Black

## PARENT SIGNATURE:
- WEEK #1:
- WEEK #2:
- WEEK #3:
- WEEK #4:
- WEEK #5:
- WEEK #6:

- NHW - No homework
- BTC - Back to class
- T - Talking
- NFD - Not following directions
- NPA - Not paying attention
- NMC - No Master Card
- MS - Misbehavior
- LC - Lost Behavior Chart
An Essay by Theresa Howard

When I have an epiphany, as I had today, I literally bubble over with joy at my newfound discovery! I did not uncover the answer to world peace. I did not find the cure for hunger in many parts of the world. So was my experience today something that changed me? For a fact, it did! As a teacher who was somewhat refreshed due to a month-long break in June, I entered Mathletes, a program designed to help teachers be better instructors of mathematics. We began our journey with a look at place value which put me in unknown territory using ORPDA, a system of counting which uses flubs and skoobrats rather than my base ten system of counting that I have been familiar with since the age of two! By unknown territory, I mean that I was totally out of my element with this new way of counting, and it scared me to death! Top that with the fact that I am approaching fifty-five, and my memory recall is not what it used to be! I felt what every child feels when they are totally uncomfortable with a concept they are trying desperately to grasp!

As the two-week Mathletes seminar progressed, we looked at the ticklish subject of long division, also known as “Do it this way because it works! Yes, I know it messes with place value in a big, bad way, but if you will just do it this way, we can move on to our next unit on geometry (or whatever is racing at us next)!” Because I was not in a traditional classroom, we stepped back and examined the process of long division more closely. It began with our instructor emphasizing the fact that we had to look at parts of the problem in “friendly numbers,” something that the student would not be afraid of. This progressed to my neighbor’s method of teaching math using an investigative approach and those “friendly numbers” again. For example, let’s look at this: 2345÷ 7=_. In the old days, someone taught me to ask myself if 7 would first go into 2; if no, then would 7 go into 23? If yes, then put a 3 over the 3 in 23 and begin the process of long division—NIGHTMARE for many students!!! Why? This absolutely will work, but it absolutely, undeniably undoes every notion students have learned about place value and what a number really stands for. What if we divided 7 into 2,345 by multiples of 10? I know that 7 times 10 equals 70 and that is not even close to my dividend. In that case, what about 7 times 100 which is equal to 700?
Now students can see that we are taking a group of 700 (7 times 100) from 2345. What remains is 1,645. If we repeat this process by taking away another group of 700 (7 times 100), we are left with a remainder of 945, and we can repeat the process again with a remainder of 245. Up to this point, we see that 2,345 can be divided by 7 - 300 times with a remainder of 245. Again, keeping with the concept of multiples of 10, we know that 7 times 10 equals 70, and what’s left is 175. Another round of 10 times 7 will leave us with a remainder of 105, and another round of 70 being taken from 105 will leave us with a remainder of 35. Let’s recap thus far: 2,345 divided by 7 will go 300 +30 times with a remainder of 35. We know that 7 times 5 is equal to 35 with no remainder. Therefore 2,345 divided by 7 = 100+100+100=10+10+10+5 or 335. Using these steps, we have watched comfortable groups of 7 that we can understand being taken from 2,345 until we are left with no remainder.

In all situations, students should begin with the familiar and what makes sense and proceed. It’s up to us, as teachers, to help the students find that which is familiar.

As time marches on, some students will realize that we can combine steps and our first amount divisible by 7 is 300 not 100+100+100. That’s what we, as teachers, should take great delight in, but the fact is if we, as students, are not comfortable with 300 and are easier with 100 at a time, we will still arrive at the correct answer in our own way that makes sense to us!! We manipulate the numbers; they don’t manipulate us!

Am I a warrior with a new cause? You bet I am! In this case, students are using a method of dividing that keeps place value intact and removes groups of numbers that begin with multiples of ten; steps that make division a series of subtracting. In all situations, students should begin with the familiar and what makes sense and proceed. It’s up to us, as teachers, to help the students find that which is familiar. Now that makes sense!
Factoring Quadratics As Easy As a, b, c
By Patricia L. Meyers

Factoring quadratics is essential in algebra. Unfortunately, many students have difficulty mastering this skill. Algebra teachers primarily teach factoring quadratics with different methods, such as guess-and-check, the Box Method, and the British Method. But there are easier methods that can simplify the process. In this paper there is an explanation of three different techniques to help factoring easier: (1) calculating the discriminant, (2) the use of a graphing calculator, and (3) the Bottoms-Up Method.

Is it Factorable? Calculate the Discriminant:

The first step before trying to factor a quadratic expression is to calculate the discriminant. If the discriminant is a perfect square, then the quadratic expression is factorable. If it is not a perfect square, then the quadratic expression is not factorable and the student only has to determine if there is a greatest common factor (GCF) to be factored out.

<table>
<thead>
<tr>
<th>Example</th>
<th>FACTORABLE</th>
<th>NOT FACTORABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x^2 - 14x + 20$</td>
<td>$x^2 + 8x - 5$</td>
<td></td>
</tr>
</tbody>
</table>

Calculate the discriminant. $b^2 - 4ac$

<table>
<thead>
<tr>
<th>Example</th>
<th>FACTORABLE</th>
<th>NOT FACTORABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = 2; b = -14; c = 20$</td>
<td>$a = 1; b = 8; c = -5$</td>
<td></td>
</tr>
<tr>
<td>$(-14)^2 - 4(2)(20) = 36$</td>
<td>$(8)^2 - 4(1)(-5) = 84$</td>
<td></td>
</tr>
</tbody>
</table>

Is the discriminant a perfect square?

<table>
<thead>
<tr>
<th>Example</th>
<th>FACTORABLE</th>
<th>NOT FACTORABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{36} = 6$</td>
<td>$\sqrt{84} \approx 9.165$</td>
<td></td>
</tr>
</tbody>
</table>

Factorable or Prime?

<table>
<thead>
<tr>
<th>Example</th>
<th>FACTORABLE</th>
<th>NOT FACTORABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factorable</td>
<td>Prime</td>
<td></td>
</tr>
</tbody>
</table>

It’s Factorable and $a = 1$! Use the Graphing Calculator: When $a = 1$, using a graphing calculator can help students who have a difficult time with factoring. Math teachers are observing more and more students who not strong with multiplication facts. Struggling students cannot determine the two factors of the constant ($c$) and the relationship to the sum of the factors ($b$). For example, the quadratic expression $x^2 - 7x +$
10, there are students who lack the mathematical skills to formulate that the two factors -2 and -5 have a product of 10 \((c)\) and a sum of -7 \((b)\). A simple technique using a graphing calculator can help these students and offer a quick way for other students to check their work.

The following is a process students can use to find the factors of a number by using a graphing calculator. For this paper, these steps incorporate the commands used on the TI-83 Plus or TI-84 graphing calculators. The process can be adapted to other hand held or internet based graphing calculators.

<table>
<thead>
<tr>
<th>ALGEBRA</th>
<th>NUMBERS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quadratic Expression</strong></td>
<td>(x^2 + bx + c)</td>
</tr>
<tr>
<td><strong>Calculate the discriminant to determine if it is a perfect square root and can (\sqrt{b^2 - 4ac})() be factored</strong></td>
<td>(b^2 - 4ac)</td>
</tr>
<tr>
<td><strong>Factorable</strong></td>
<td>(\sqrt{36} = 6)</td>
</tr>
<tr>
<td><strong>Calculator</strong></td>
<td>Use the function key:</td>
</tr>
<tr>
<td>(Y=)</td>
<td>(Y=)</td>
</tr>
<tr>
<td><strong>Type the formula next to (Y_1 =).</strong></td>
<td>(c \div X_{T, S, n})</td>
</tr>
<tr>
<td><strong>Generate a table.</strong></td>
<td>2ND</td>
</tr>
</tbody>
</table>
Evaluate table for two integers whose product is equal to $c$ and whose sum equals $b$.

Looking for two integers whose product is $c$ and sum is $b$:

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$q$</td>
</tr>
<tr>
<td>$s$</td>
<td>$t$</td>
</tr>
<tr>
<td>$m$</td>
<td>$n$</td>
</tr>
<tr>
<td>$c$</td>
<td>$j$</td>
</tr>
<tr>
<td>$n$</td>
<td>$m$</td>
</tr>
</tbody>
</table>

$m \cdot n = n \cdot m = c \\
 m + n = n + m = b$

Looking for two integers whose product is $-8$ and sum is $2$:

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$4$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$8$</td>
</tr>
<tr>
<td>$0$</td>
<td>ERROR</td>
</tr>
<tr>
<td>$1$</td>
<td>$-8$</td>
</tr>
<tr>
<td>$2$</td>
<td>$-4$</td>
</tr>
<tr>
<td>$3$</td>
<td>$-2.667$</td>
</tr>
<tr>
<td>$4$</td>
<td>$-2$</td>
</tr>
</tbody>
</table>

$-2 \cdot 4 = 4 \cdot -2 = -8 \\
-2 + 4 = 4 + -2 = 2$

Write the quadratic expression in factored form.

$$(x + m)(x + n)$$

$$(x - 2)(x + 4)$$

It’s Factorable, but ! Use the Bottoms-Up Method:

A method of factoring quadratics not familiar to many teachers is called the Bottoms-Up Method. It is an easy method for students to follow, but the user must be cautious to factor out the greatest common factor (GCF) or a negative if the leading coefficient is less than zero (this will be explained in the next section). Following is how to use the Bottoms-Up Method using algebra and with an example.
<table>
<thead>
<tr>
<th>Quadratic Expression</th>
<th>ALGEBRA</th>
<th>NUMBERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(ad)x^2 + (bd)x + (cd)$</td>
<td>$12x^2 + 2x - 4$</td>
<td></td>
</tr>
<tr>
<td>Calculate the discriminant to determine if it is a perfect square root and can be factored.</td>
<td>$(bd)^2 - 4(ad)(cd)$</td>
<td>$2^2 - 4(12)(-4) = 196$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sqrt{196} = 14$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Factorable</td>
</tr>
<tr>
<td>Determine if there is a GCF. If there is one, factor it out, but do not drop it.</td>
<td>$(ad)x^2 + (bd)x + (cd)$</td>
<td>$(2 \times 6)x^2 + (2 \times 1)x - (2 \times 2)$</td>
</tr>
<tr>
<td>Multiply the leading coefficient ($a$) to the constant ($c$). Drop the leading coefficient for now, but it will be used later.</td>
<td>$d(x^2 + bx + ac)$</td>
<td>$2(x^2 + x - 6 \times 2)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2(x^2 + x - 12)$</td>
</tr>
<tr>
<td>Factor the new trinomial. The graphing calculator can be used at this point to determine the two factors.</td>
<td>Let $b = m + n$ and $c = mn$</td>
<td>$b = 1 = 4 + (-3)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c = -12 = (4)(-3)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d(x + m)(x + n)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2(x + 4)(x - 3)$</td>
</tr>
<tr>
<td>Divide each binomial’s constant by the leading coefficient ($a$).</td>
<td>$d(x + \frac{m}{a})(x + \frac{n}{a})$</td>
<td>$a = 6$</td>
</tr>
<tr>
<td>Simplify the fractions in the binomials.</td>
<td>$\frac{m}{a} = \frac{p}{q}$ and $\frac{n}{a} = \frac{r}{s}$</td>
<td>$2(x + \frac{2}{3})(x - \frac{1}{2})$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d(x + \frac{p}{q})(x + \frac{r}{s})$</td>
</tr>
</tbody>
</table>
In each binomial move any remaining denominators to be the coefficient of the linear term.

<table>
<thead>
<tr>
<th></th>
<th>(d(qx + p)(sx + r))</th>
<th>(2(3x + 2)(2x - 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Check result.</td>
<td>(d(qx + p)(sx + r))</td>
<td>(= 2(3x + 2)(2x - 1))</td>
</tr>
<tr>
<td></td>
<td>(= (ad)x^2 + (bd)x + (cd))</td>
<td>(= 2(6x^2 - 3x + 4x - 1))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(= 2(6x^2 + x - 1))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(= 12x^2 + 2x - 4)</td>
</tr>
</tbody>
</table>

**Caution About the Bottoms-Up Method:**

There are two things that must be remembered about the Bottoms-Up Method. First of all, the GCF must be factored out first. The second caution is when the leading coefficient is negative, then a negative GCF must be factored out. Two examples have been provided to show how the improper use of the Bottoms-Up Method will result in an incorrect factorization of a quadratic.
<table>
<thead>
<tr>
<th>Quadratic Expression</th>
<th>EXAMPLE WITH GCF</th>
<th>EXAMPLE WITH A NEGATIVE LEADING CO-</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$6x^2 + 28x + 30$</td>
<td>$-4x^2 + 4x + 3$</td>
</tr>
<tr>
<td>Calculate the discriminant to determine if it is a perfect square root and can be factored.</td>
<td>$28^2 - 4(6)(30) = 64$</td>
<td>$4^2 - 4(-4)(3) = 64$</td>
</tr>
<tr>
<td></td>
<td>$\sqrt{64} = 8$</td>
<td>$\sqrt{64} = 8$</td>
</tr>
<tr>
<td></td>
<td>Factorable</td>
<td>Factorable</td>
</tr>
<tr>
<td>Determine if there is a GCF. If there is one, factor it out.</td>
<td>OOPS! The GCF of 2 is not factored out.</td>
<td>OOPS! The GCF of $-1$ is not factored out.</td>
</tr>
<tr>
<td>Multiply the leading coefficient ($a$) with the constant ($c$). Drop the leading coefficient for now, but it will be used later.</td>
<td>$x^2 + 28x + 6 \times 30$</td>
<td>$x^2 + 4x + (-4) \times 3$</td>
</tr>
<tr>
<td></td>
<td>$x^2 + 28x + 180$</td>
<td>$x^2 + 4x - 12$</td>
</tr>
<tr>
<td>Factor the new trinomial. The graphing calculator can be used at this point to determine the two factors.</td>
<td>$(x + 18)(x + 10)$</td>
<td>$(x + 6)(x - 2)$</td>
</tr>
<tr>
<td>Divide each binomial’s constant by the coefficient of the $x^2$ term ($a$).</td>
<td>$(x + \frac{18}{6})(x + \frac{10}{6})$</td>
<td>$(x + \frac{6}{-4})(x - \frac{2}{-4})$</td>
</tr>
<tr>
<td>Simplify the fractions in the binomials.</td>
<td>$(x + 3)(x + \frac{5}{3})$</td>
<td>$(x - \frac{3}{2})(x + \frac{1}{2})$</td>
</tr>
</tbody>
</table>
In each binomial move the any remaining denominators to be the coefficient of the linear term.

<table>
<thead>
<tr>
<th></th>
<th>((x + 3)(3x + 5))</th>
<th>((2x - 3)(2x + 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x + 3)(3x + 5))</td>
<td>((2x - 3)(2x + 1))</td>
<td></td>
</tr>
<tr>
<td>(3x^2 + 5x + 9x + 5)</td>
<td>(4x^2 + 2x - 6x - 3)</td>
<td></td>
</tr>
<tr>
<td>(3x^2 + 14x + 5)</td>
<td>(4x^2 - 4x - 3)</td>
<td></td>
</tr>
<tr>
<td>(\neq 6x^2 + 28x + 30)</td>
<td>(\neq -4x^2 + 4x + 3)</td>
<td></td>
</tr>
<tr>
<td>Not the original expression.</td>
<td>Not the original expression.</td>
<td></td>
</tr>
</tbody>
</table>

**Conclusion:**

There are many methods teachers and students can use to factoring a quadratic. The techniques described in this paper can be used independently, with each other, and with other methods. The key is students should be provided with many methods to factoring quadratics and allowing them to achieve the correct results. The goal of mathematics is to calculate the correct answer by using sound mathematical techniques. Calculating the discriminant, the use of a graphing calculator, and the Bottoms-Up Method are all techniques which can be used to help reach the correct factorization of a quadratic.
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