# **Chapter 4.** Integration

## **4.1.** Indefinite Integrals, Differential Equations, and Modeling

**Definition.** A function $F(x)$ is an *antiderivative* of a function $f(x)$ if $F'(x) = f(x)$ for all $x$ in the domain of $f$. The **set** of all antiderivatives of $f$ is the *indefinite integral* of $f$ with respect to $x$, denoted by $\int f(x)\,dx$. The symbol $\int$ is an *integral sign.* The function $f$ is the *integrand* of the integral, and $x$ is the *variable of integration.*

**Note.** We denote the indefinite integral SET as

$$\int f(x)\,dx = F(x) + C$$

where $F$ is a specific antiderivative and $C$ represents an "arbitrary constant." (In class, we will use "$k$" for a specific constant.)

**Examples.** Page 319 numbers 12 and 23.

**Table 4.1.** Integral Formulas

| Indefinite Integral | Derivative Formula |
|---|---|
| **1.** $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$, $n$ rational | $\frac{d}{dx}\left[\frac{x^{n+1}}{n+1}\right] = x^n$ |
| $\int dx = \int 1\,dx = x + C$ (special case) | $\frac{d}{dx}[x] = 1$ |
| **2.** $\int \sin kx\,dx = -\frac{\cos kx}{k} + C$ | $\frac{d}{dx}\left[-\frac{\cos kx}{k}\right] = \sin kx$ |
| **3.** $\int \cos kx\,dx = \frac{\sin kx}{k} + C$ | $\frac{d}{dx}\left[\frac{\sin kx}{k}\right] = \cos kx$ |
| **4.** $\int \sec^2 x\,dx = \tan x + C$ | $\frac{d}{dx}[\tan x] = \sec^2 x$ |
| **5.** $\int \csc^2 x\,dx = -\cot x + C$ | $\frac{d}{dx}[-\cot x] = \csc^2 x$ |
| **6.** $\int \sec x \tan x\,dx = \sec x + C$ | $\frac{d}{dx}[\sec x] = \sec x \tan x$ |
| **7.** $\int \csc x \cot x\,dx = -\csc x + C$ | $\frac{d}{dx}[-\csc x] = \csc x \cot x$ |

**Definition.** The problem of finding a function $y$ of $x$ when we know $y'$ and a value of $y$ at a particular point $x_0$ is called an *initial value problem.*

**Example.** Page 321 numbers 54 and 58, page 320 number 36.