More Chapter 3

Projectile motion simulator

http://www.walter-fendt.de/ph11e/projectile.htm

The equations of motion for constant acceleration from chapter 2 are valid separately for both motion in the x and y directions. However the initial velocity now has 2 component (x and y components).

If projectile has an initial velocity at an angle θ_0 with respect to the horizontal axis, θ_0 is known as the projection angle. The components of the initial velocity are:

$v_{0x} = v_0 \cos \theta_0$ $\quad\quad$ $v_{0y} = v_0 \sin \theta_0$

Equations of motion

rewriting the equations of motion for each direction we get:

$v_x = v_{0x} + a_x t$

$\Delta x = v_{0x} t + ½\, a_x t^2$

$v_x^2 = v_{0x}^2 + 2a_x \Delta x$

$v_y = v_{0y} + a_y t$

$\Delta y = v_{0y} t + ½\, a_y t^2$

$v_y^2 = v_{0y}^2 + 2a_y \Delta y$

Where $v_{0x} = v_0 \cos \theta_0$

θ_0 is known as the projection angle

Where $v_{0y} = v_0 \sin \theta_0$

Equations of motion

for projectiles only under influence of gravity

$$a_x = 0 \qquad a_y = -g$$

$v_x = v_{0x}$

$\Delta x = v_{0x} t$

$v_x^2 = v_{0x}^2$

$v_y = v_{0y} - g\,t$

$\Delta y = v_{0y} t - \frac{1}{2} g\, t^2$

$v_y^2 = v_{0y}^2 - 2g\Delta y$

Where $v_{0x} = v_0 \cos\theta_0$

θ_0 is known as the projection angle

Where $v_{0y} = v_0 \sin\theta_0$

Conclusions

Because $a_x = 0$, a projectile's velocity in the x-direction is a constant. ($v_x = v_{0x}$)

<u>The vertical component of the velocity and the vertical displacement are identical to those of a free falling object</u>.

<u>Projectile motion can be described as a superposition of two independent motions</u>.

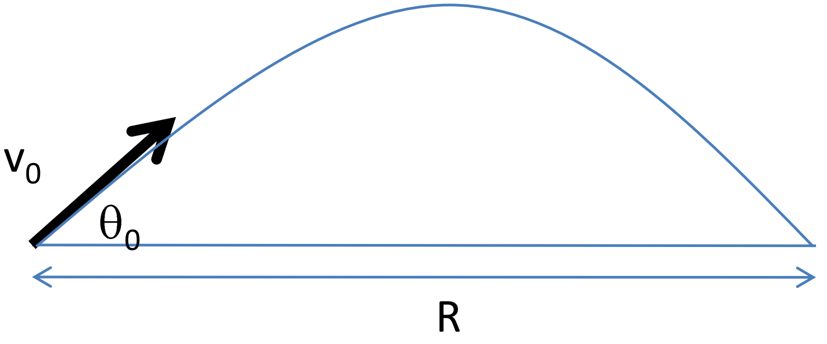
Motion diagram for projectile motion.

See figure 3.16 on page 64.

Range

Another useful equation gives the range of a projectile for a given initial velocity and projection angle.

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$



This only works when the launch and landing points are at the same elevation.

Figure 3.15 pg 63

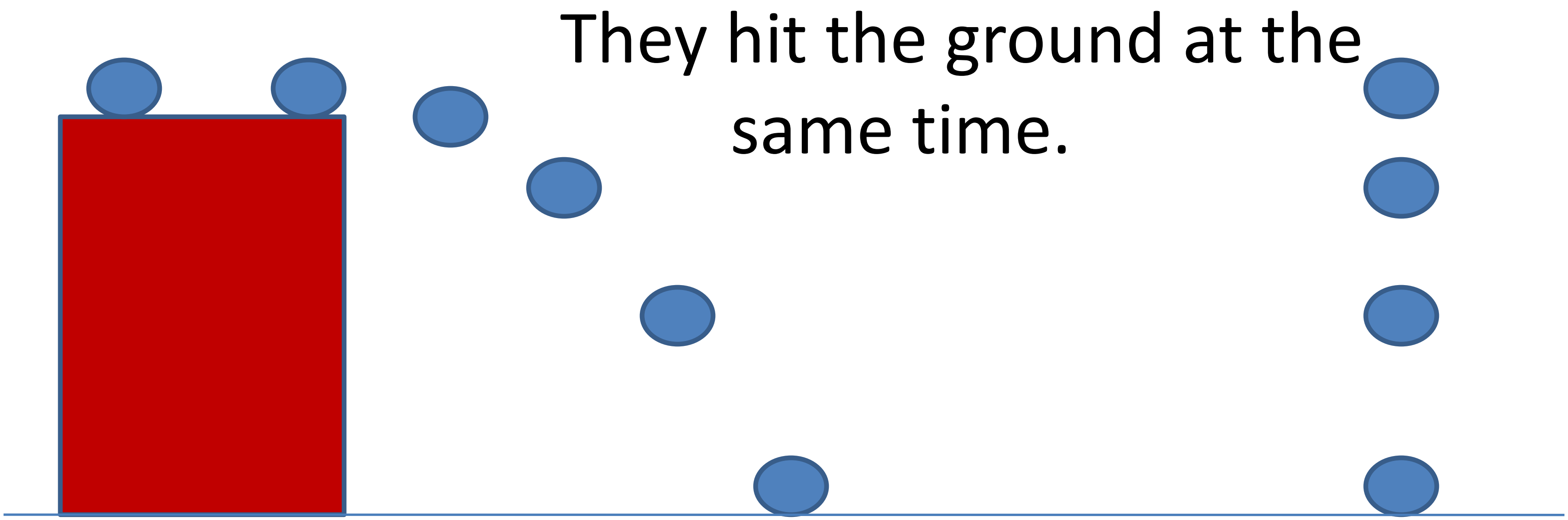
Shows the trajectories of an object shot with the same initial velocity but at different projection angles.

Maximum range occurs when θ_0 = 45 degrees.

45^0 is the angle that maximizes $\sin(2\theta)$

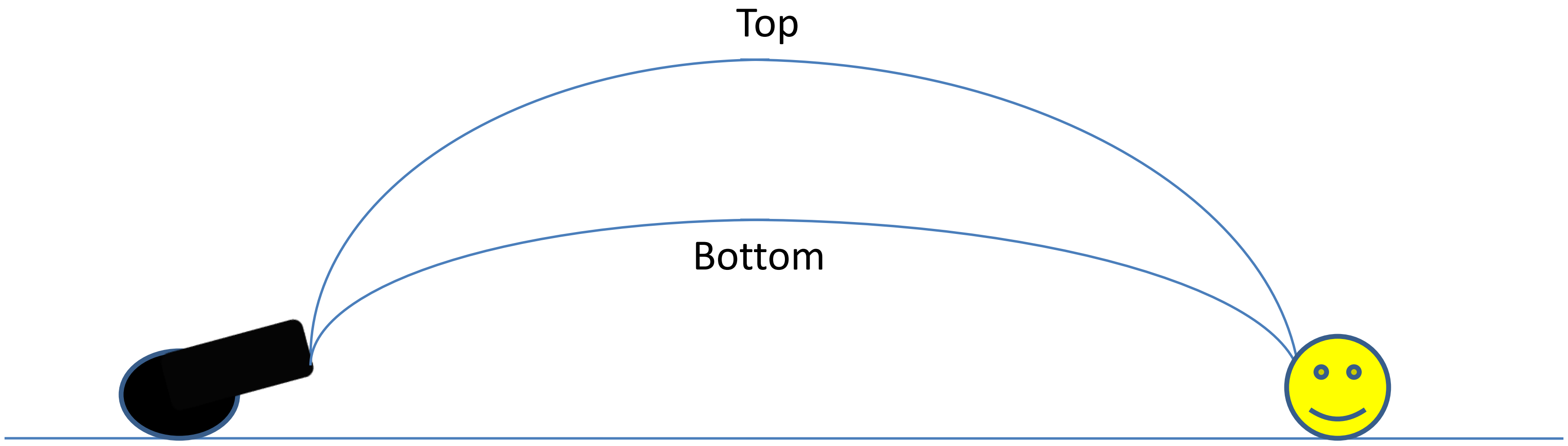
Because the horizontal motion is constant, while the vertical motion is accelerated, it will be the motion in the vertical direction that will determine how long a projectile is in the air.

The time that a projectile is in the air depends on the vertical displacement and the vertical component of velocity. Here we roll a ball off a table and drop a ball at the same time.

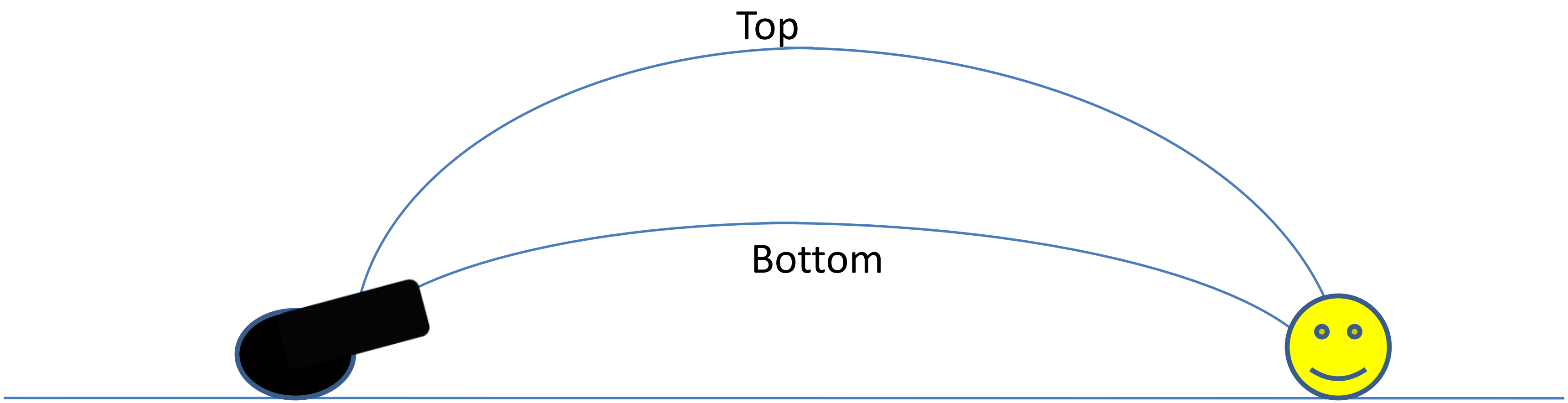


Two Cannons

Which cannonball hits the target first?



Two Cannons



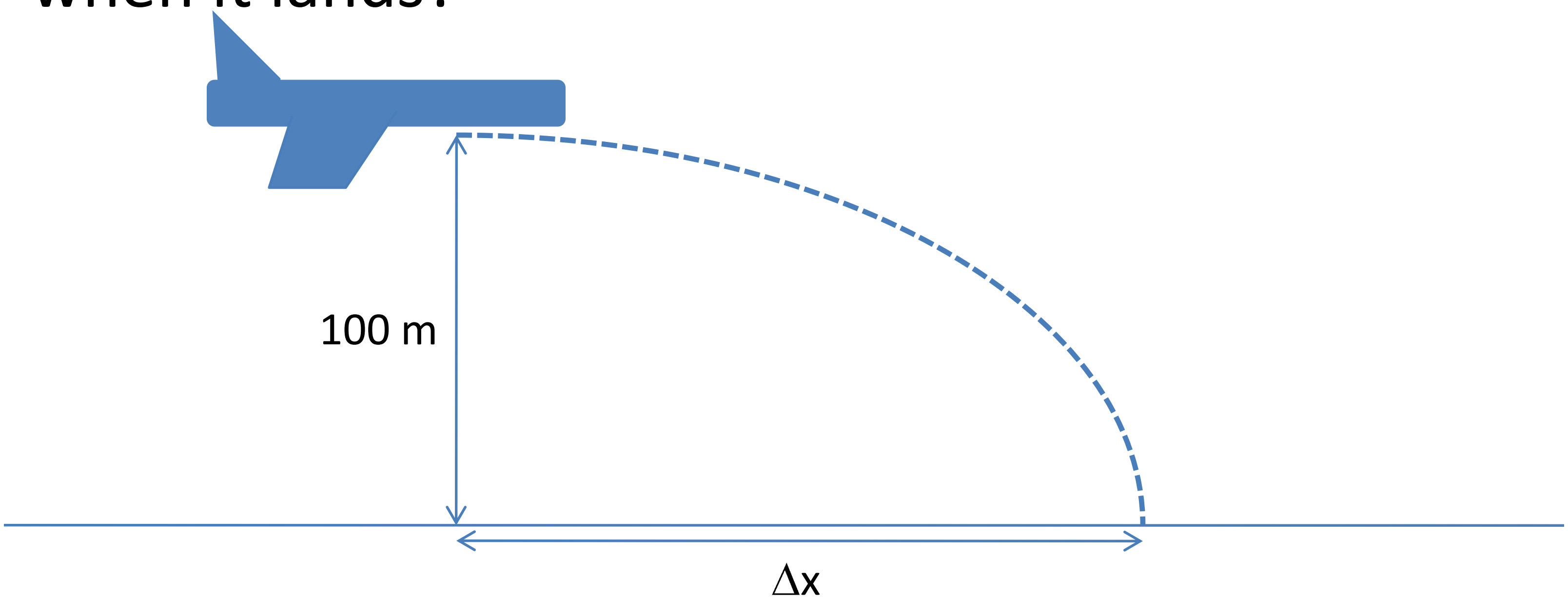
The cannonball that follows the bottom path hits first. Both travel the same horizontal distance. But the cannonball taking the bottom path needs a higher x-velocity to make the shot. Since the horizontal distances are the same but the lower cannonball has a higher x-velocity, the lower cannonball arrives first.

Another website to play around on.

http://hyperphysics.phy-astr.gsu.edu/hbase/traj.html

Examples

Rescue plane drops a package while the plane is traveling horizontally at 40.0 m/s, 100 meters above the ground. What is the horizontal distance the package travels while falling? What are the horizontal and vertical components of the velocity when it lands?



We know:

v_{0x} = 40 m/s v_{0y} = 0 m/s y_0 = 100 m

Want to know horizontal distance package travels.

First find the time the package is in the air.

(Use the y direction)

$\Delta y = y - y_0 = v_{0y}t - ½ g t^2$

$0 \text{ m} - 100 \text{ m} = - ½ g t^2$ solve for time

$t = \sqrt{\frac{100\,m * 2}{9.8 m/s^2}} = 4.5 \text{ s}$

Now do $x = v_x t$ $x = (40 \text{ m/s})(4.5 \text{ s}) = 181 \text{ m}$

Find the horizontal and vertical components of the velocity when the package lands.

Horizontal component is constant v_{fx} $= v_{ox} = 40$ m/s

Want vertical component of velocity:

use: $v_y = v_{0y} – g\,t$

$v_y = 0 – (9.8\ m/s^2)(4.5\ s) = -44.1\ m/s$

Could also use $v_y^2 = v_{0y}^2 – 2g\Delta y$

$v_y^2 = -\ 2(9.8\ m/s^2)(-100\ m)$

$v_y = 44.3\ m/s$

this method didn’t give the minus sign for direction.

Cannonball example.

A cannon can propel a cannonball with a velocity of 150 m/s. If the projectile is shot at an angle of 30 degrees, how far will it travel? (assuming it lands at same level it is shot)

Fastest way to solve is to use range equation.

$$R = \frac{v_0^2 \sin(2\theta)}{g} = \frac{(150\,m/s)^2 \sin(2*30)}{9.8m/s^2}$$

R = 1988 m

We could also find how far the cannonball goes by:

Finding the time the cannonball is in the air.

Then use: $x = v_{0x}t$

To find time ball is in air, use: $\Delta y = y - y_0 = v_{0y}t - ½\, g\, t^2$

$\Delta y = 0 = (150\ m/s)(\sin 30)\, t - ½\, (9.8\ m/s^2)\, t^2$

This simplifies to: $(150\ m/s)(\sin 30) = ½\, (9.8\ m/s^2)\, t$

Solving for time gives t = 15.3 seconds.

$x = v_{0x}t$ (150 m/s) (cos 30) (15.3 s) = 1988 m

How much farther will the cannon shoot it the projection angle is 45 degrees?

R = (150 m/s)^2 sin (2*45)/(9.8 m/s^2) = 2296 m

What about 60 degrees?

R = (150 m/s)^2 sin (2*60)/(9.8 m/s^2) = 1988 m

What about 90 degrees?

R = (150 m/s)^2 sin (2*90)/(9.8 m/s^2) = 0 m

Long Jumper

A long jumper leaves the ground at a 20 degree angle and has a velocity of 11 m/s.

a)how long does it take for him to reach maximum height?

At maximum height the vertical component of velocity is zero.

Use $v_y = v_{0y} – g\ t$

$0 = (11\ m/s)(\sin 20) – g\ t$

$t = (11\ m/s)(\sin 20)/(9.8\ m/s^2)$

$t = 0.38\ s$

b) What is the maximum height?

Plug in the time it takes to get to maximum height.

$y_{max} = \Delta y = v_{0y}t - ½\ g\ t^2$

$y_{max} = (11\ m/s)(\sin 20)(0.38\ s) - ½\ (9.8\ m/s^2)(0.38\ s)^2$

$y_{max} =$ 0.72 m

d) Or using equation 3.14 c in text

$v_y^2 = v_{0y}^2 - 2g\Delta y$

$\Delta y = (v_y^2 - v_{0y}^2)/(2\ g) = \dfrac{0 - (11 m/s * \sin(20))^2}{-2(9.8 m/s^2)}$

$\Delta y =$ 0.72 m

c) How far does he jump?

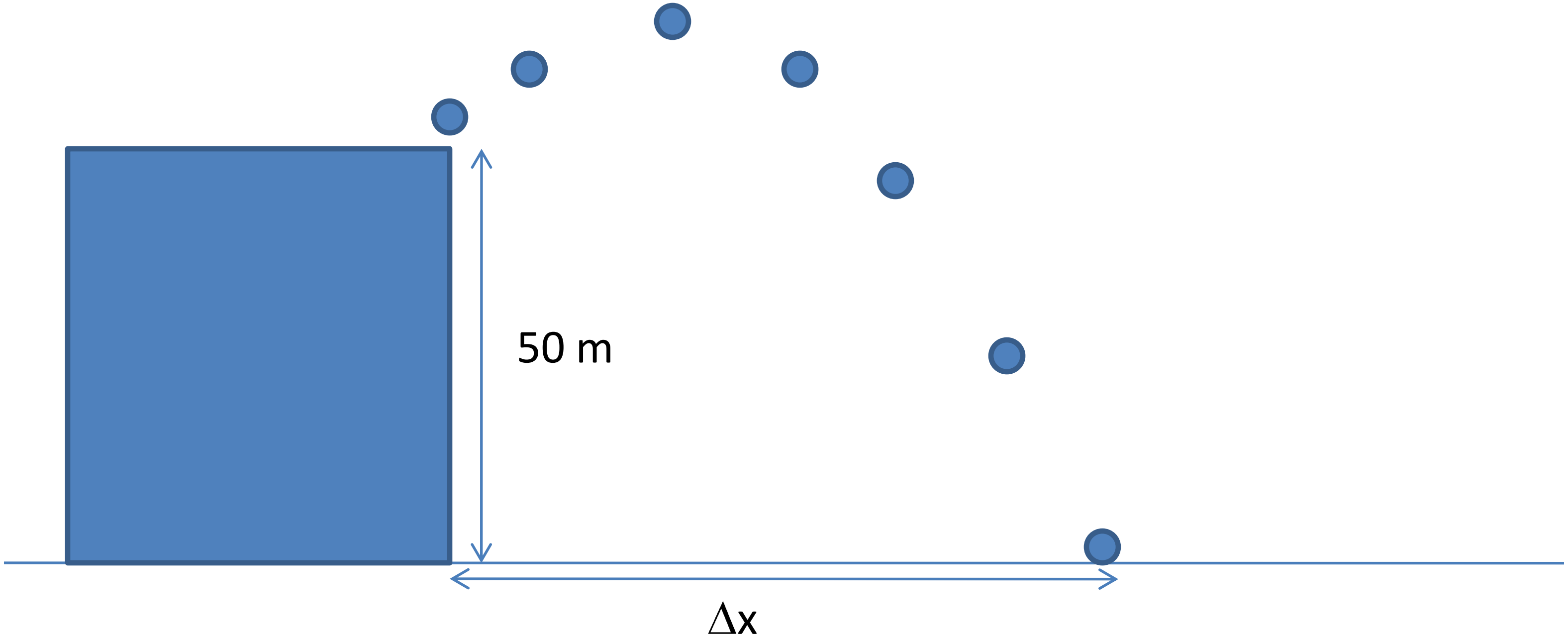
$\Delta x = v_{0x}\, 2t_{max}$ = (11 m/s)(cos 20)*2(0.38s)

Δx = 7.9 m

Jumper is in the air for twice the time it takes to reach maximum height.

Throwing a ball from rooftop

Suppose you throw a ball from a 50 m tall building, with a velocity of 30 m/s and a projection angle of 30 degrees. How far from the building's base, will the ball land?



Find the time the ball is in the air.

$\Delta y = v_{0y}t - ½\, g\, t^2$

$0\ m - 50\ m = (30\ m/s)(\sin 30)\ t - ½\ (9.8\ m/s^2)t^2$

$-50\ m = (15\ m/s)\ t - (4.9\ m/s^2)\ t^2$

Solve this quadratic equation for t gives us:

t = -2.0 and 5.1 seconds

Pick the positive value

Now use the horizontal velocity and time to get the horizontal distance traveled.

$x = v_{0x}t = (30\ m/s)(\cos 30)\ (5.1\ s) = 133\ m$.

Rocket

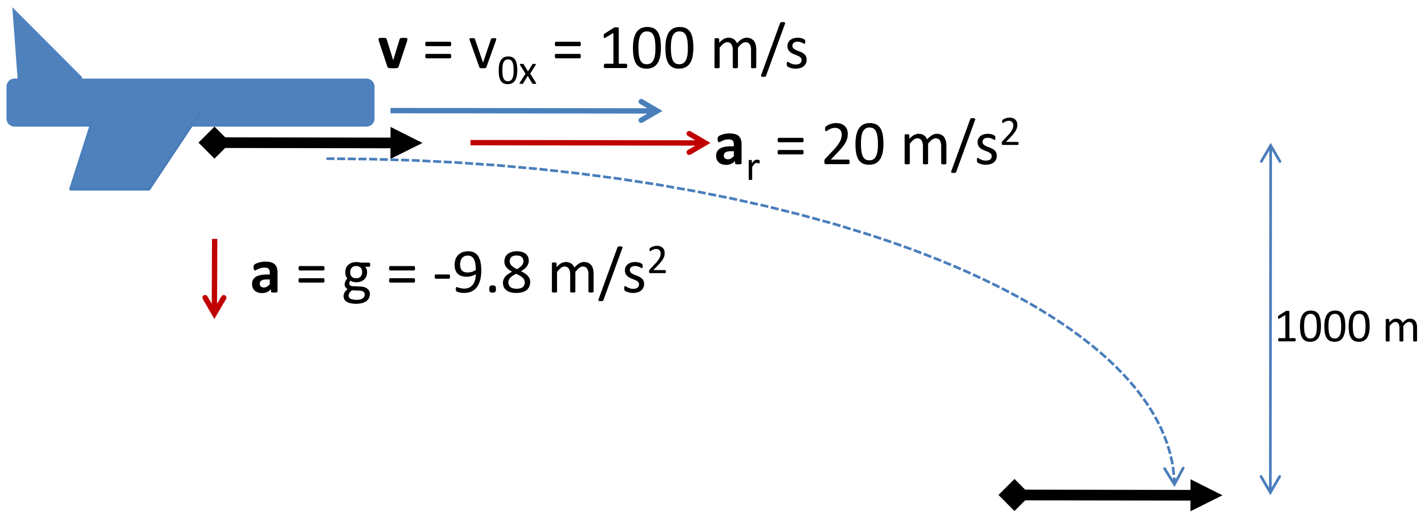
Jet plane traveling horizontally at 100 m/s drops a rocket from a high altitude. The rocket immediately fires its engines and accelerates 20 m/s^2 in the x-direction, while it falls due to gravity in the y-direction. After rocket has fallen 1 km, find:

a) the y-velocity

b) x-velocity

c) magnitude and direction of the velocity

Rocket



a) Finding the y-velocity

Initially the rocket is moving only in the x-direction. So $v_{0y} = 0$

We know the vertical distance fallen is 1 km, but not the time.

use: $v_y^2 = v_{0y}^2 - 2g\Delta y$

$v_y^2 = 0^2 - 2(9.8\ \mathrm{m/s^2})(-1000\ \mathrm{m})$

$v_y =$ 140 m/s downward

We could have also found the y-velocity by the following method:

find time needed to fall 1000 m

$\Delta y = v_{0y}t + \frac{1}{2} a_y t^2$

$\Delta y = y_f - y_0 = -1000\text{m} = \frac{1}{2}(-9.8\text{m/s}^2)t^2$

$t = \sqrt{\dfrac{1000\,m}{4.9 m/s^2}} = 14.3\text{ s}$

then do $v_{fy} = v_{0y} + a\,t$

$v_{fy} = 0 - (9.8\text{m/s}^2)(14.3\text{ s}) = -140\text{ m/s}$

The negative sign means down

b) Find the x-velocity.

$v_{0x} = 100$ m/s

$a_x = a_r = 20$ m/s^2

time is 14.3 seconds

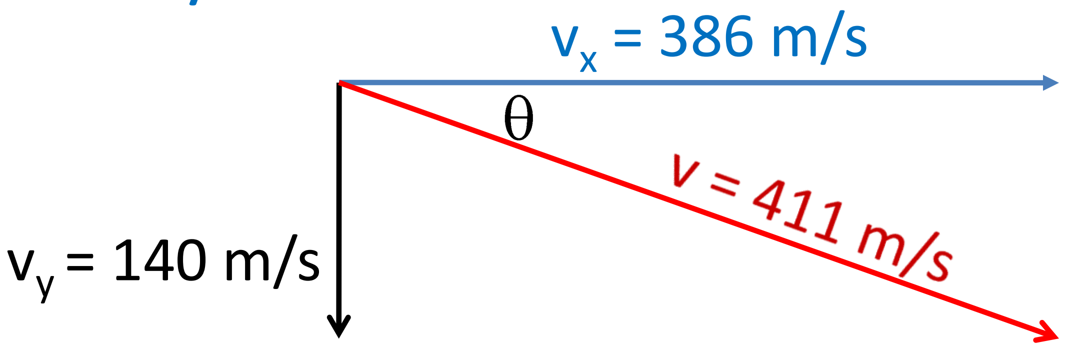
Use $v_{fx} = v_{0x} + a_x t = 100\text{m/s} + (20\text{m/s}^2)(14.3\text{ s})$

$v_{fx} = 386$ m/s in x-direction

c) find the magnitude and direction of the velocity of the rocket after falling 1 km.

To get the magnitude use the velocity's components and apply $\sqrt{v_x^2 + v_y^2}$.

v= $\sqrt{386^2 + 140^2}$ = 411 m/s



to get θ, use $\tan^{-1}(140/386) = 20^0$

Below horizontal axis

Problem 22. Baseball pitched at 101 mph is thrown horizontally. How far would the ball fall vertically by the it reached home plate, 60.6 feet away.

First convert to mks units.

$$\frac{101mi}{1hr}\frac{1609m}{1mi}\frac{1hr}{3600s} = \frac{101\times1609m}{3600s} = 45.1m/s$$

$$60.5\,ft\,\frac{12in}{1\,ft}\frac{2.54cm}{1in}\frac{1m}{100\,cm} = \frac{60.5\times12\times2.54m}{100} = 18.4m$$

v_{0x} = 45.1 m/s v_{0y} = 0 a = -g Δx = 18.4 m

First, use the horizontal velocity and the horizontal distance traveled to find the time the ball is pitched.

Use $\Delta x = v_{0x}t$ rewritten as: $t = \Delta x/v_{0x}$

$t = (18.4 \text{ m})/(45.1 \text{ m/s}) = 0.41 \text{ s}$

Now use this time and apply to a falling body to get the vertical distance the ball falls.

$\Delta y = v_{0y}t + ½ a_y t^2 = ½ (-9.8 \text{m/s}^2)(0.41 \text{ s})^2$

$\Delta y = - 0.82 \text{ m}$

Negative because the ball drops.

Projectile Motion Problem Solving Strategy

Remember that the horizontal and vertical components of motion are independent.

However they will both occur over the same time interval.

Use information from one component of the motion to determine the time the projectile is in the air.

Apply this time to the other component of the motion.

- Useful tip:

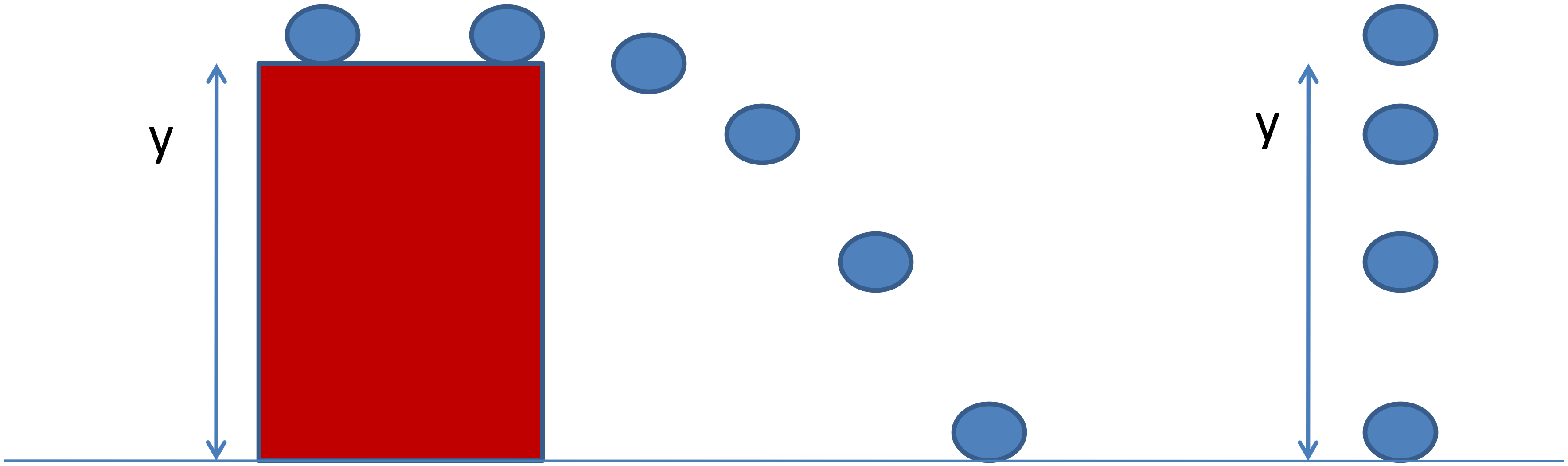
  When a projectile ***has no initial velocity in the vertical direction***, the time the object takes to fall a distance y, is:

  $y = \frac{1}{2} g t^2$

  $y = \frac{1}{2} (9.8\ m/s^2) t^2$

  $t = \sqrt{2y/g}$

The two balls hit the ground at the same time.



Person falls out of a window that is 15 meters above the ground. The time spent falling is:

t = $\sqrt{2y/g}$ = $\sqrt{\dfrac{2\times 15m}{(9.8m/s^2)}} = 1.75s$

For a 440 m fall: $t = \sqrt{\dfrac{2\times 440\,m}{9.8m/s^2}} = 9.5s$