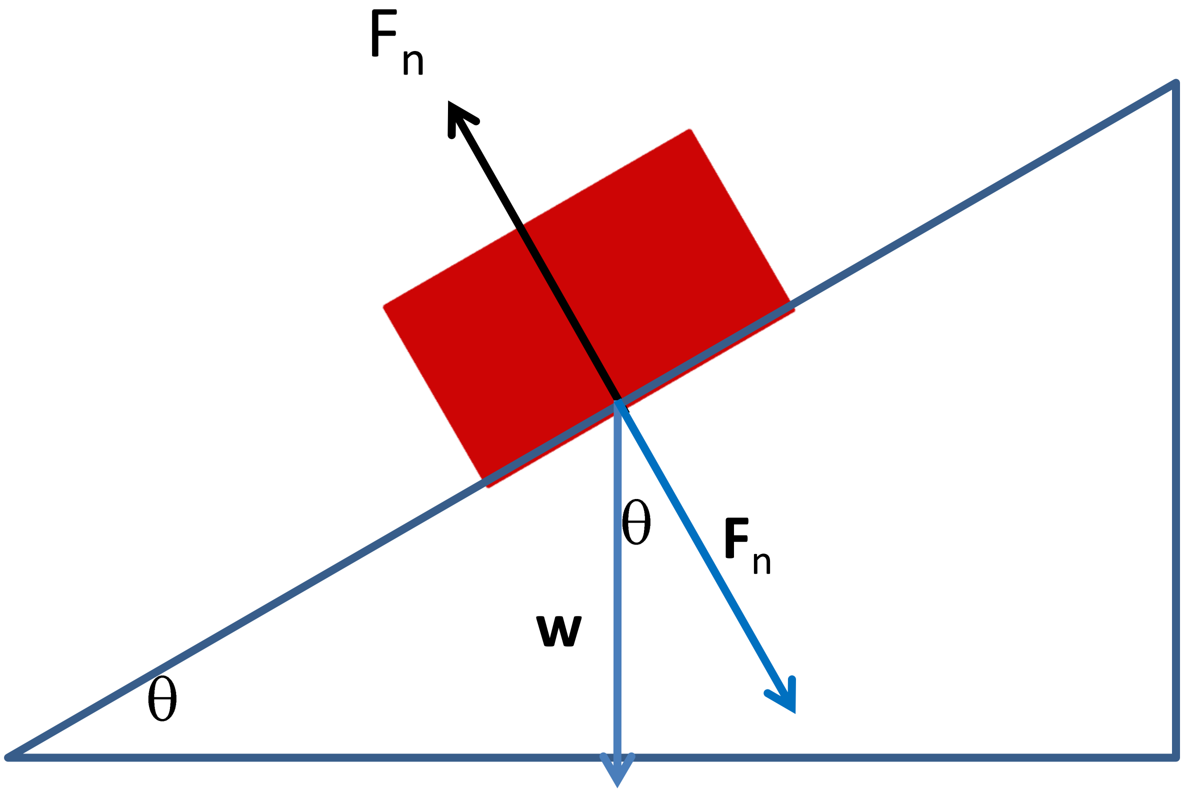
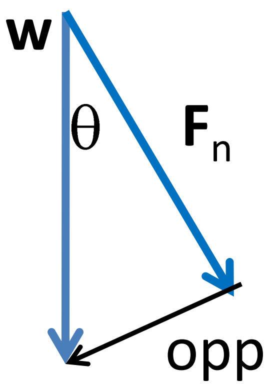
More Ch 4 Forces

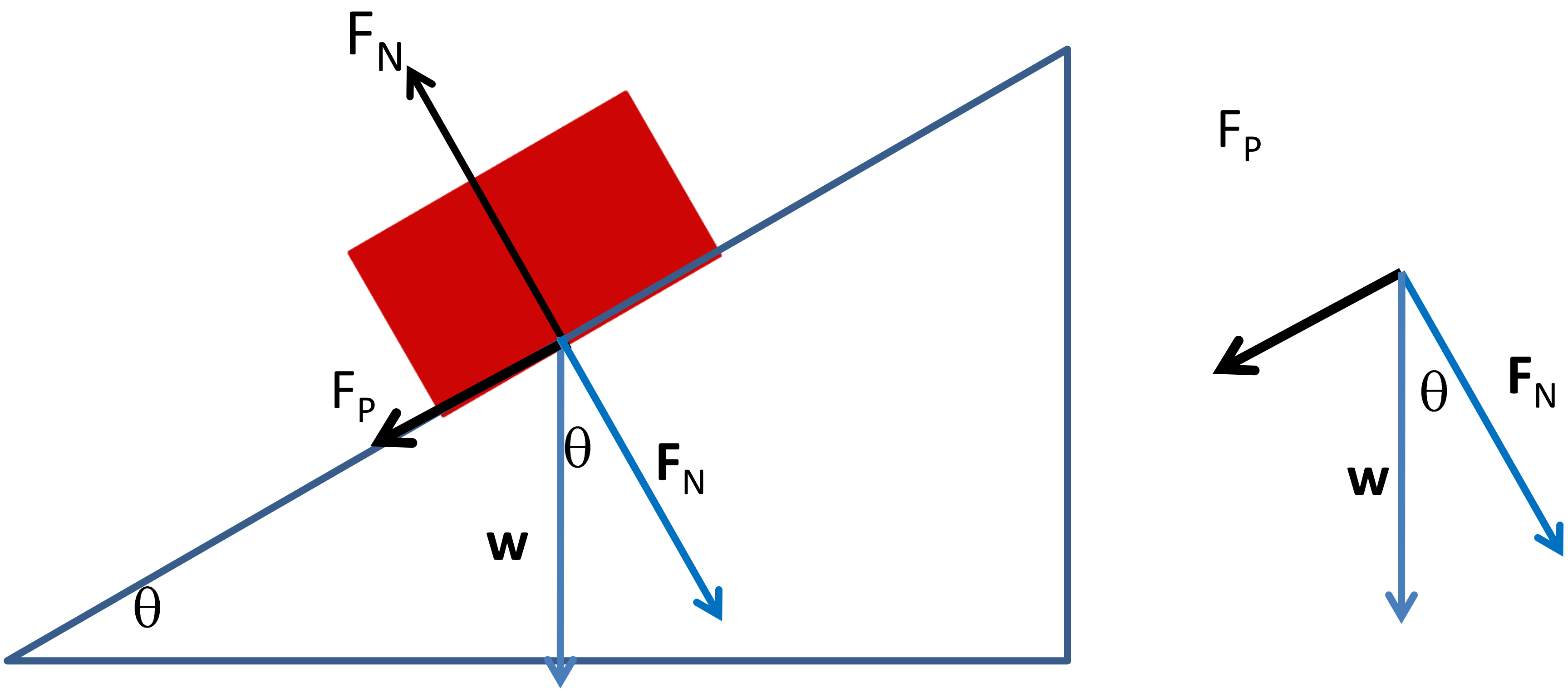
Do quick quiz 4.5

Weight of Object on Inclined Plane





- F_n is the adjacent component of the weight.
- What about the opposite component?



F_P, is the opposite (parallel) component of the weight vector. It is parallel to the inclined plane. This is the component of the weight that causes the object to slide down the plane.

$$F_P = W \sin\theta = mg \sin\theta$$

What happens when we increase θ from 0 to 90 degrees? Let mass = 50 kg.

$F_P = (50\text{ kg})(9.8\text{m/s}^2) \sin 0 = 0$

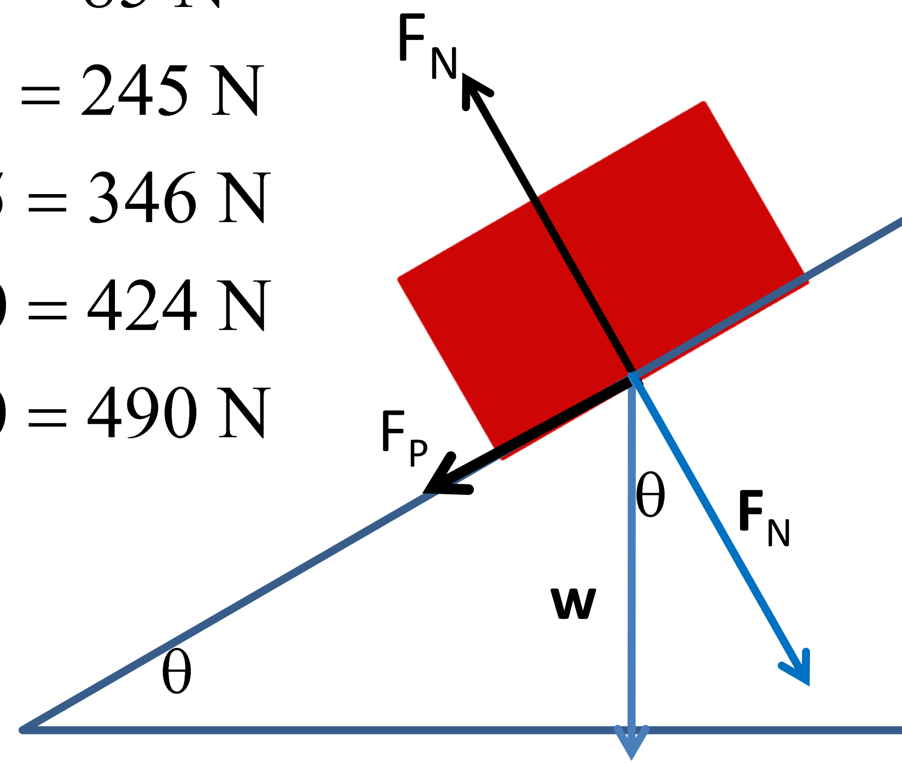
$F_P = (50\text{ kg})(9.8\text{m/s}^2) \sin 10 = 85\text{ N}$

$F_P = (50\text{ kg})(9.8\text{m/s}^2) \sin 30 = 245\text{ N}$

$F_P = (50\text{ kg})(9.8\text{m/s}^2) \sin 45 = 346\text{ N}$

$F_P = (50\text{ kg})(9.8\text{m/s}^2) \sin 60 = 424\text{ N}$

$F_P = (50\text{ kg})(9.8\text{m/s}^2) \sin 90 = 490\text{ N}$



Normal Forces (from Tuesday)

What happens when we increase θ from 0 to 90 degrees?

$F_N = (50\ kg)(9.8 m/s^2) \cos\theta$

$F_N = (50\ kg)(9.8 m/s^2) \cos 0 = 490\ N$

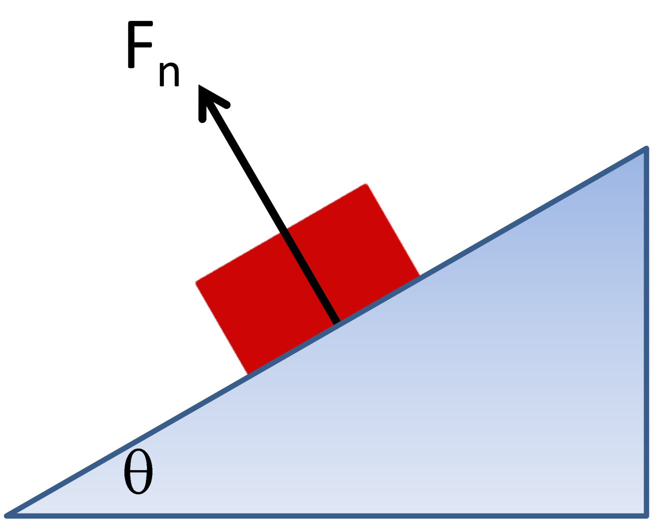
$F_N = (50\ kg)(9.8 m/s^2) \cos 10 = 483\ N$

$F_N = (50\ kg)(9.8 m/s^2) \cos 30 = 424\ N$

$F_N = (50\ kg)(9.8 m/s^2) \cos 45 = 346\ N$

$F_N = (50\ kg)(9.8 m/s^2) \cos 60 = 245\ N$

$F_N = (50\ kg)(9.8 m/s^2) \cos 90 = 0\ N$



As the angle of inclination increases:

F_N decreases

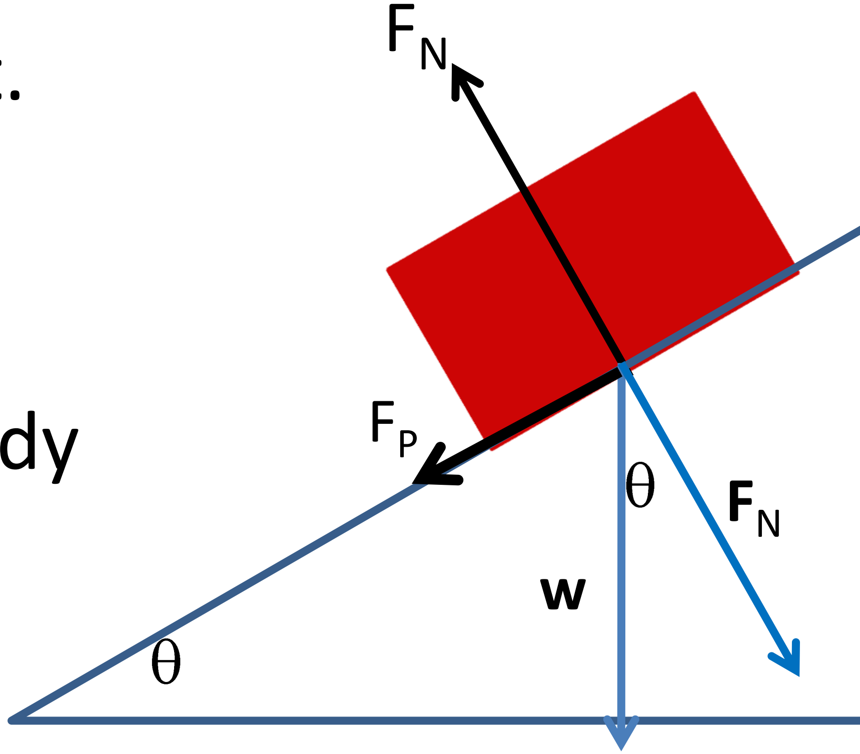
F_P increases

As the slope increases, gravity has a greater effect on the object.

When incline is vertical,

Object resembles falling body

F_P = weight = mg



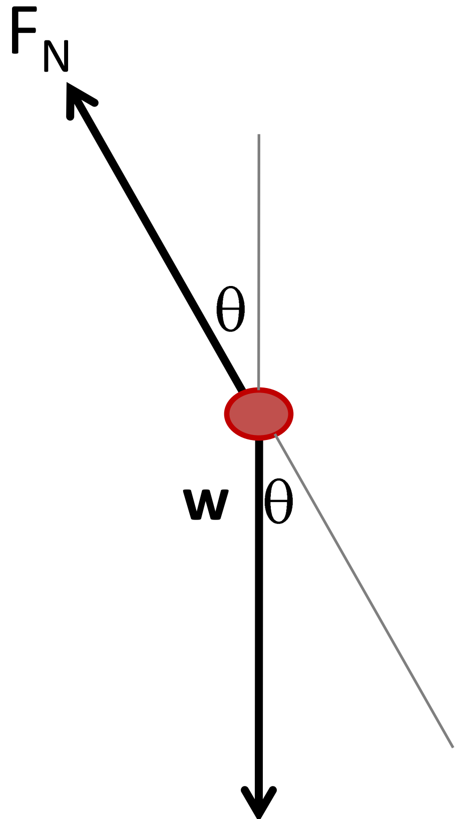
What acceleration will the block have?

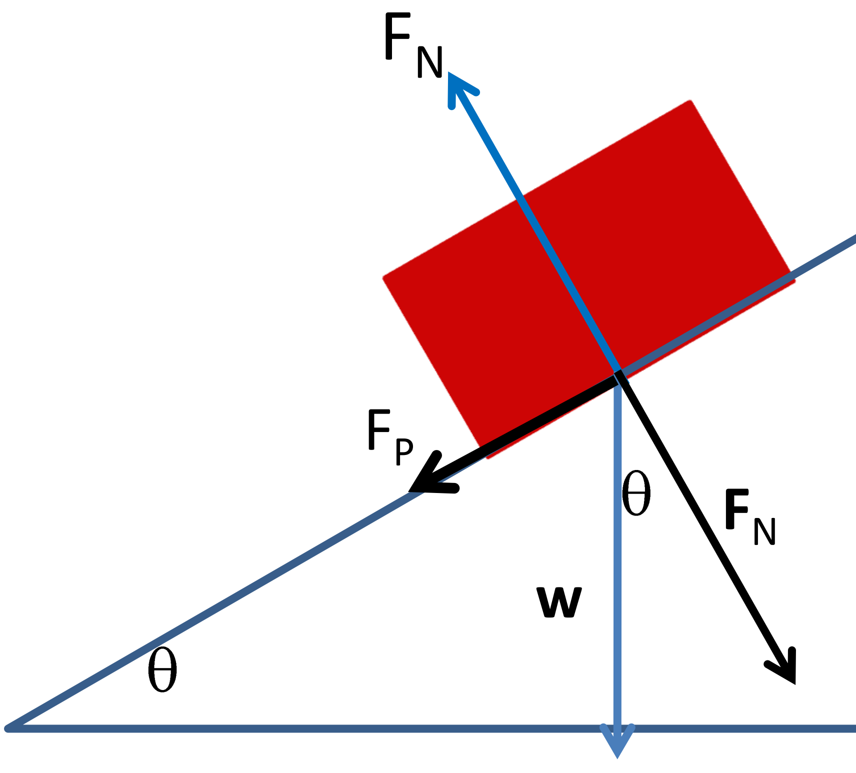
Let: $m = 50$ kg, $\theta = 30^0$

The block is free to accelerate along the incline.

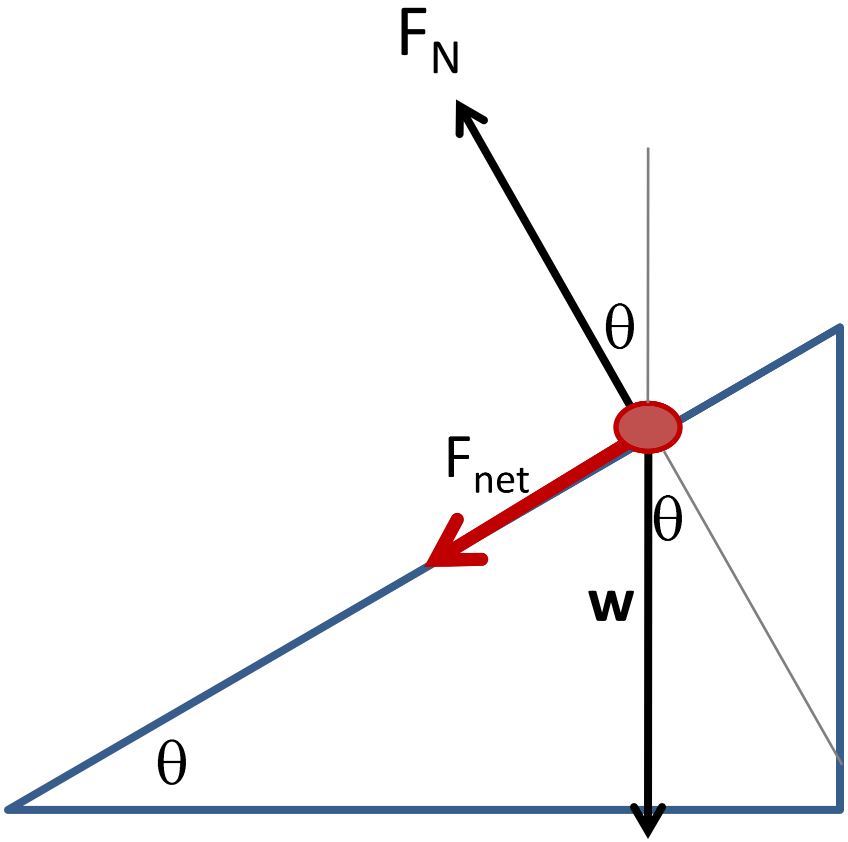
So F_P is the accelerating force.

Free body diagram:





Free body diagram



F_{net} is the total, or resultant force. It is directed along the plane.

Applying the 2nd Law

F_{net} = ma = (50kg)a

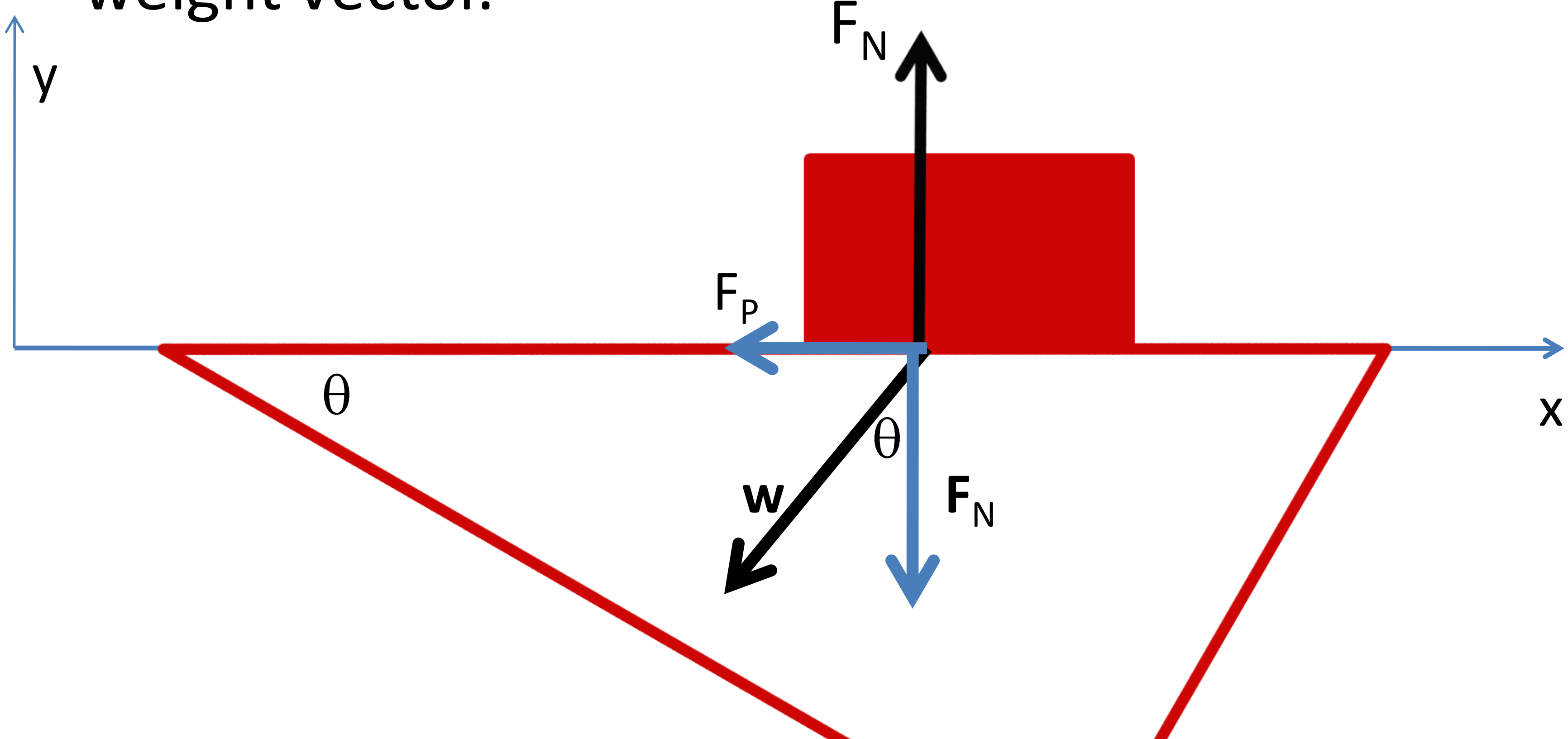
a = F_{net}/m

a = (50kg)g (sin 30)/(50 kg)

a = 4.9 m/s^2

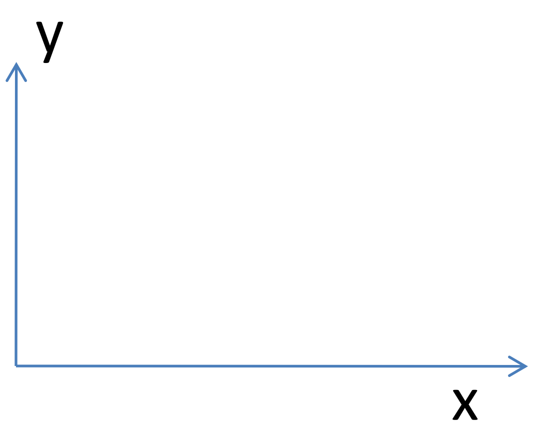
Another way of looking at incline plane problems (rotating the coordinate system)

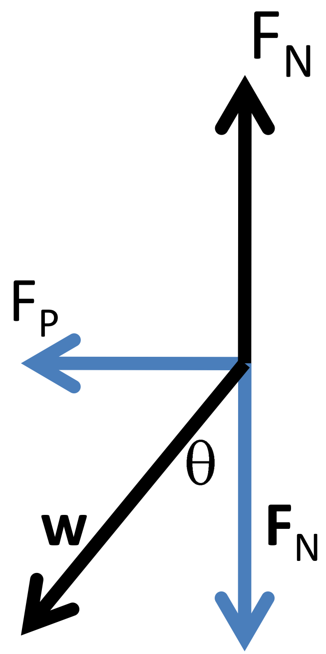
Rotate coordinates by angle of incline, θ. Now the normal force is along the y-direction. You can see the x and y (parallel and normal components of the weight vector.



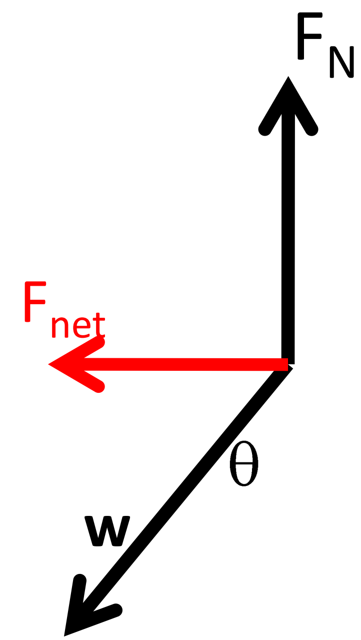
Free Body diagram

The normal component of the weight is again balanced out by the normal force. F_{net} is the parallel component of the weight.





Vector addition

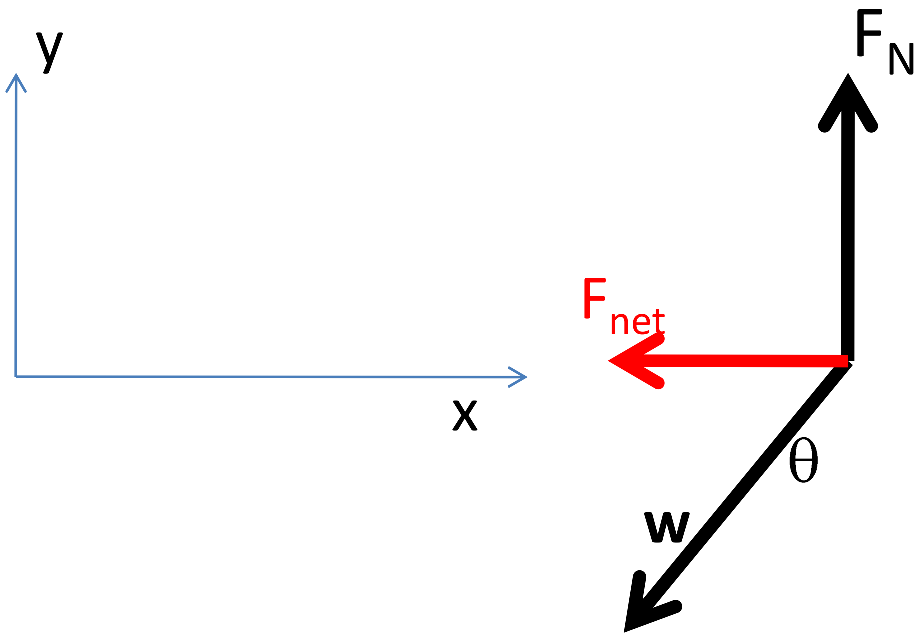


Again solving for acceleration

$a = F_{net}/m$

$a = mg(\sin\theta)/m = g\sin\theta = (9.8 m/s^2)\sin 30 = 4.9 m/s^2$

Vector addition

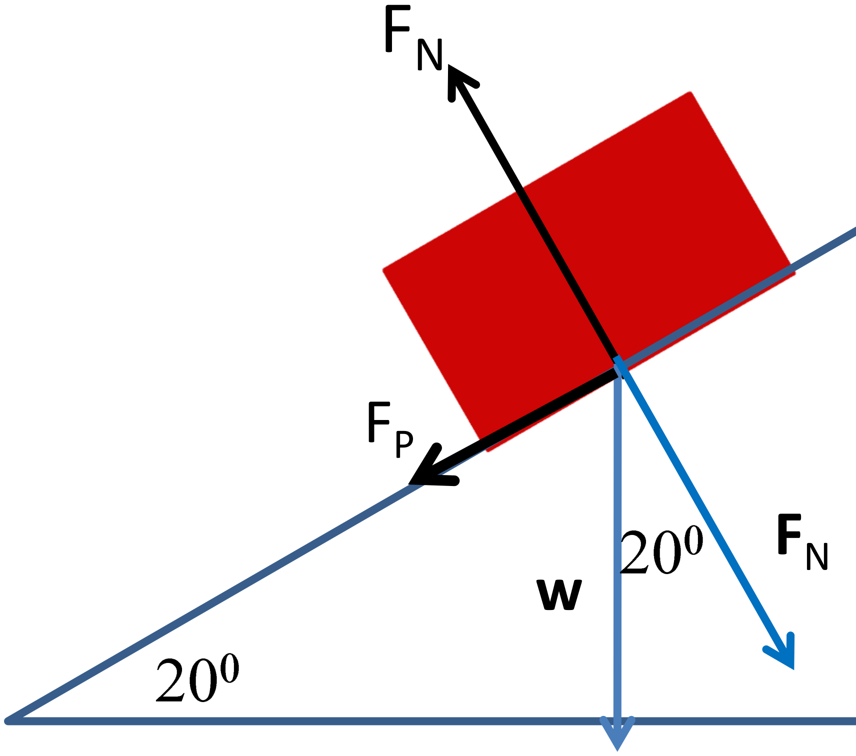


This tells us that the acceleration down an incline plane is equal to gravity times the inclination angle

$$a = g\sin\theta$$

Example

A 2 kg box is slides down a frictionless incline plane $\theta = 20^0$. The block starts from rest. After the block moves 2 meters, what will its velocity be?



First we use the force work to find:

F_{net} = ma a = $\frac{F_{net}}{m} = \frac{mg(\sin\theta)}{m}$

a = g sin θ

a = g sin 20 = 3.4m/s^2

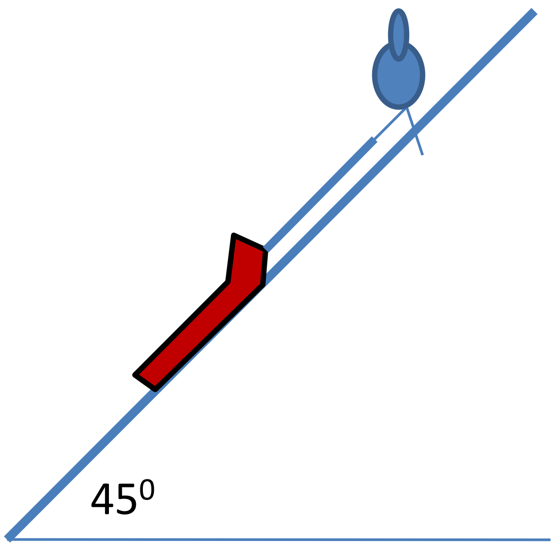
Now use the acceleration and solve for the velocity of the block after sliding 2 meters.

$v_f^2 = v_0^2 + 2a\Delta x$ = (0 m/s)^2+ 2(3.4m/s^2)(2m)

V_f = 3.7 m/s (direction is down the incline)

What force (PULL)is needed to pull a sled up a frictionless hill at constant velocity?

- mass of sled = 10 kg
- Angle of hill is 45 degrees.



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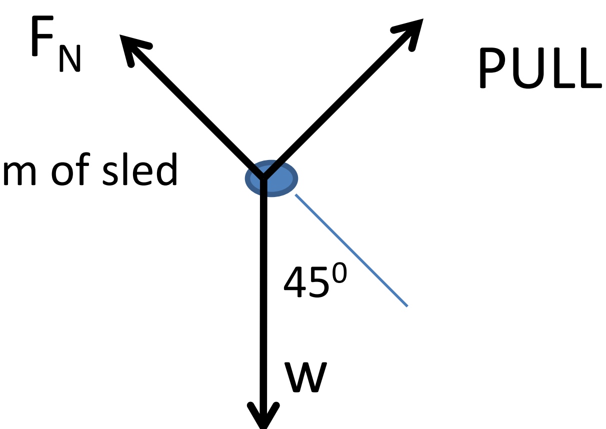
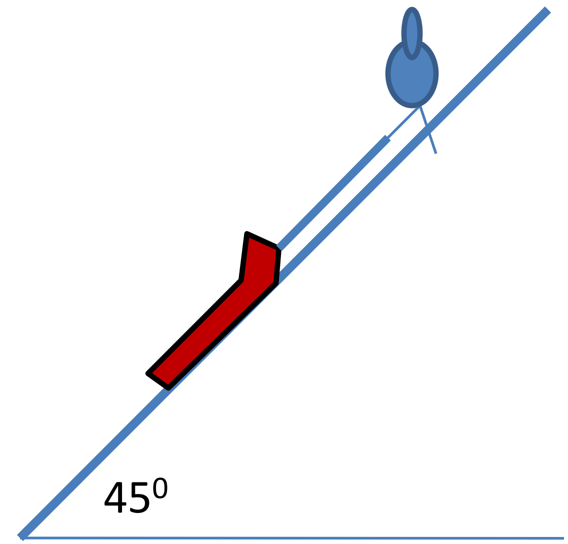


Free body diagram of sled



Since the velocity is constant, the sum of all the forces needs to be zero. So the PULL force needs to balance out the parallel component of the weight vector.

PULL = mg (sin 45) = (10kg)g (sin 45) = 69 N

Friction

- Friction is due to surfaces not being perfectly smooth.
- Friction force ALWAYS opposes the direction of motion.
- The result of friction forces is energy (heat)
- Two types of friction:
  - Static friction force keeps the object at rest, and is needed to be overcome so the object can begin motion.
  - Kinetic friction occurs when one object is sliding against another surface.

Friction

When is friction used is ordinary life?

- car brakes
- makes car move
- driving around a turn
- walking
- rubbing your hands together to make warmth

Friction

How to calculate frictional force.

Frictional force, F_f , depends on the material of the two surfaces involves. Given by coefficient of friction (μ). See table on page 103 for examples.

There are two coefficients: static (μ_s) and kinetic (μ_k)

F_f also depends on the normal force

Static friction force $F_f \leq \mu_s F_N$

Kinetic friction force $F_f = \mu_k F_N$

Static friction

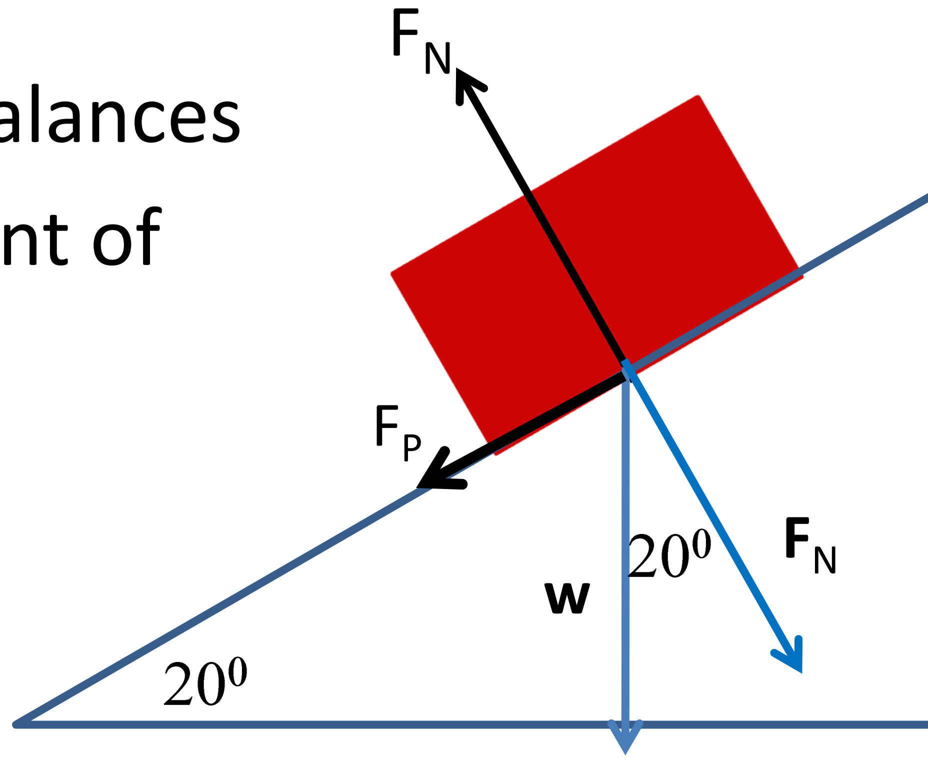
- $F_f \leq \mu_s F_N$
- The static friction force can vary from 0 to $\mu_s F_N$.
- $\mu_s F_N$ is known as the maximum static friction force.
- This is the force needed to be overcome to start sliding an object across the surface.

- See figure 4.19 on page 102.

Example

What coefficient of friction is needed to keep a block from sliding down an incline?

The static friction force balances out the parallel component of The weight.

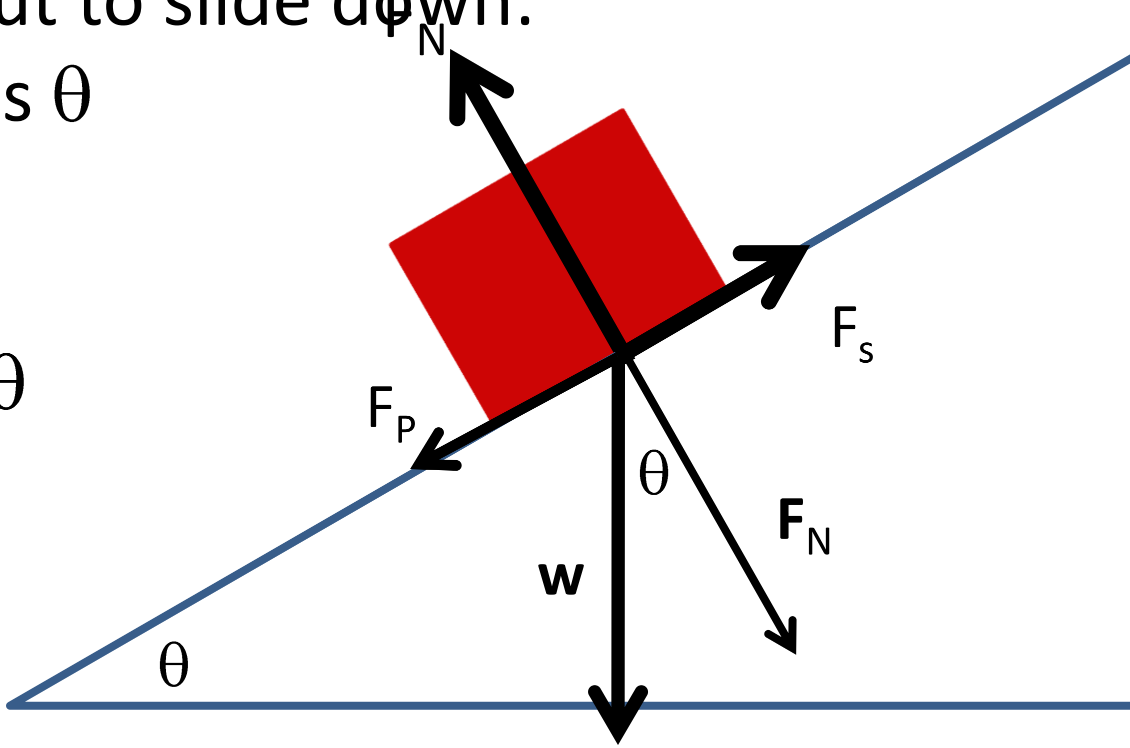


F_s balances out F_P

From earlier we know $F_P = mg \sin \theta$

When the box is about to slide down:

$F_f = \mu_s F_N = \mu_s mg \cos \theta$



Therefore:

$\mu_s mg \cos \theta = mg \sin \theta$

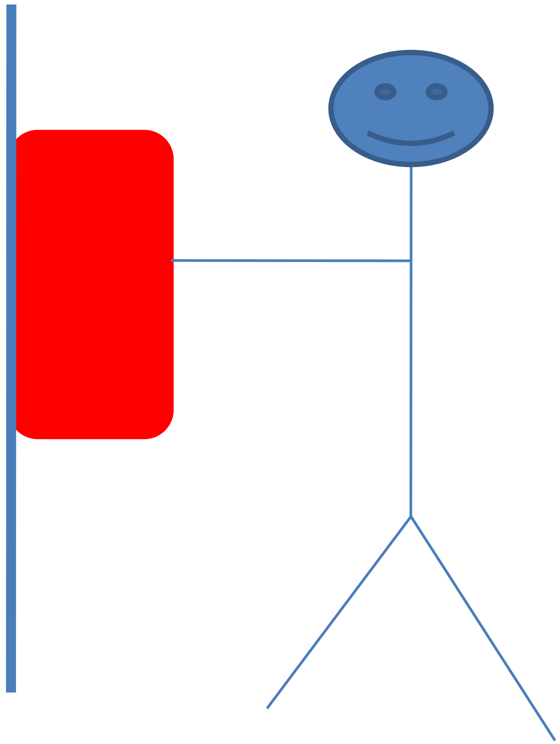
Solve for μ_s gives:

$\mu_s = \tan \theta$

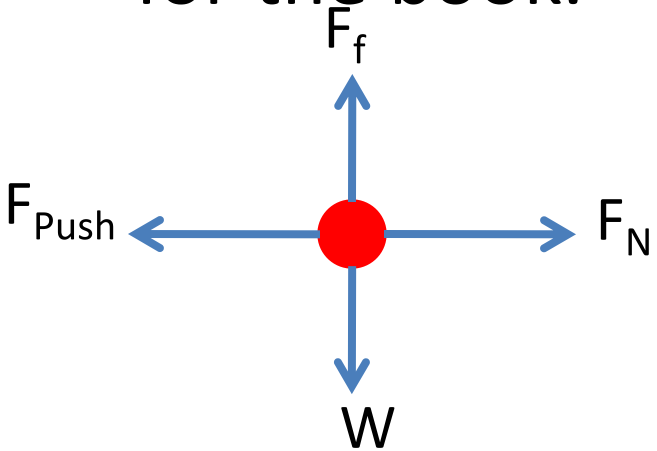
This tells us the static coefficient, μ_s, needed to keep an object from sliding down an incline depends only on the incline.

Do quick quizzes 4.6, 4.7, 4.8 on page 103

In this example you are holding a book still, up against the wall. How hard do you have to push to keep the book from sliding?

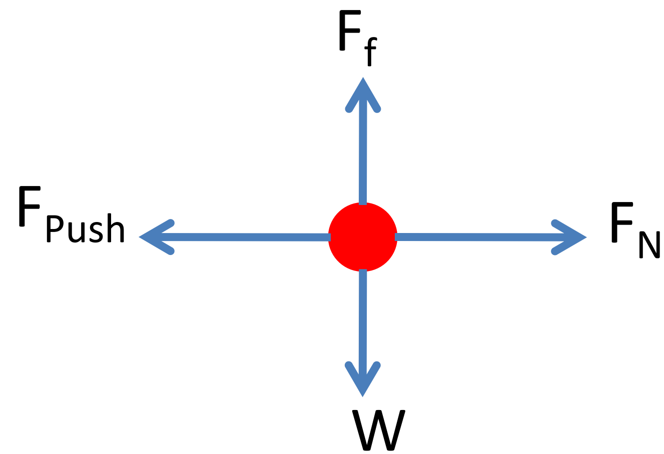


Free body diagram for the book.



If the book is not accelerating, the sum of the forces is zero.

F_N and F_{push} counteract each other in the horizontal direction.



Weight and friction force balance each other out in the vertical direction. The friction is what is holding up the book.

Let the book have mass 5 kg and $\mu_s = .4$.

$W = (5kg)g = 49$ N

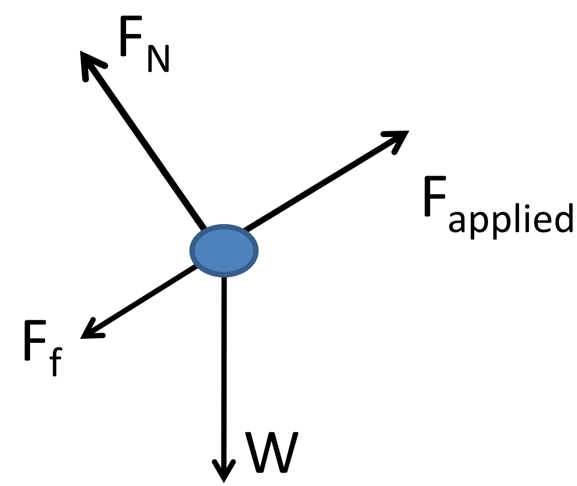
For $W = F_f$, $F_f = \mu_s F_N = 0.4F_N = 0.4\ F_{push}$

$49\text{ N} = 0.4\ F_{push}$ $\quad F_{push} = 122.5\text{ N}$

Kinetic Friction Problem

You are pushing a block up an incline. The angle of inclination is 30 degrees. The mass of the block is 20 kg. The coefficients of friction are μ_s = 0.3 and μ_k = 0.2. What force must be applied to the box to keep the speed constant?

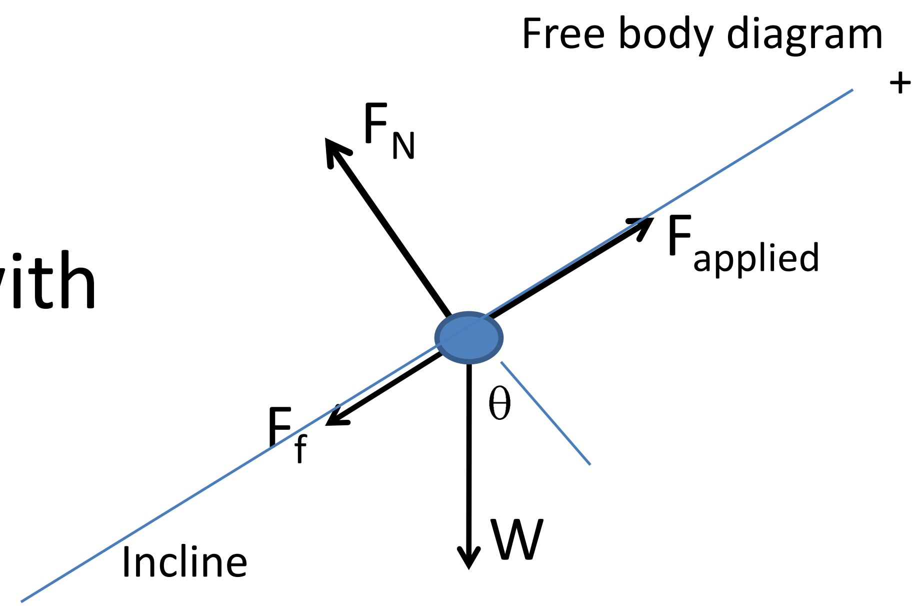
Free body diagram



Since block is sliding across surface use μ_k = 0.2.

Since velocity is constant, a = 0.

$F_{net} = ma = 0$



We are only concerned with motion along the incline.

$F_{applied}$, F_f, and W have components along the incline.

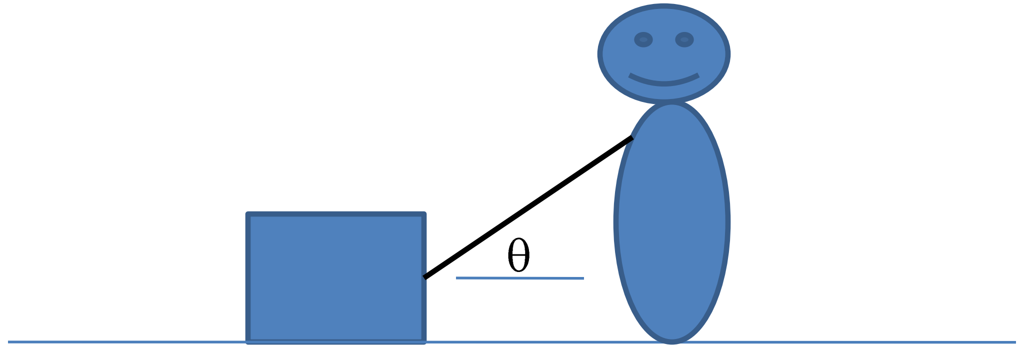
Writing 2nd Law along incline we get:

$F_{applied} - F_f - mg \sin 30 = ma = 0$

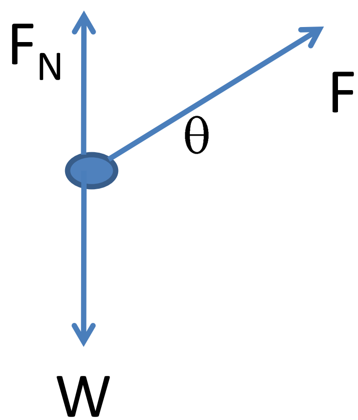
$F_{applied} = F_f + mg \sin 30 = \mu_k mg(\cos 30)+mg(\sin 30)$

$F_{applied} = 0.2(20kg)g(\cos 30) + (20kg)g(\sin 30) = 131$ N

Pulling with force (F) on box at an angle



Free Body Diagram



2nd Law equations give us:

X-direction: $F \cos \theta = m\, a_x$

Y-direction: $F \sin \theta + F_N - W = m\, a_y = 0$

$F_N = W - F \sin \theta$

Normal force is reduced.

You hang a mass from a scale.

When you jerk the scale upwards, what does the scale read?

When you accelerate the scale downward, what does it read?

When you jerk the scale upwards, the reading will increase.

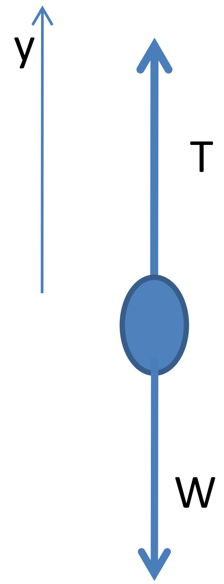
When you jerk the scale down, the reading decreases.

Jerking the scale upwards:

The force exerted by the scale (T) on the mass is in the positive y-direction.

W is in the negative y-direction.

Acceleration is in the positive y-direction.



2nd Law: $\Sigma F = ma$

$$T - W = ma$$

$$T = W + ma$$

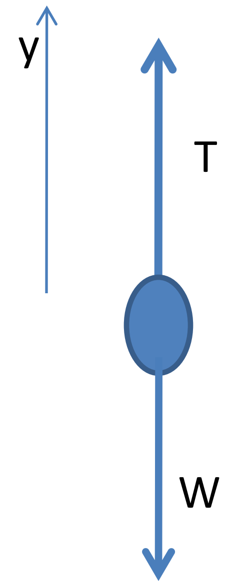
$$T = mg + ma = m(g+a)$$

Jerking the scale downwards:

The force exerted by the scale (T) on the mass is in the <u>positive</u> y-direction.

W is in the <u>negative</u> y-direction.

Acceleration is in the <u>negative</u> y-direction.



2^{nd} Law: $\Sigma F = ma$

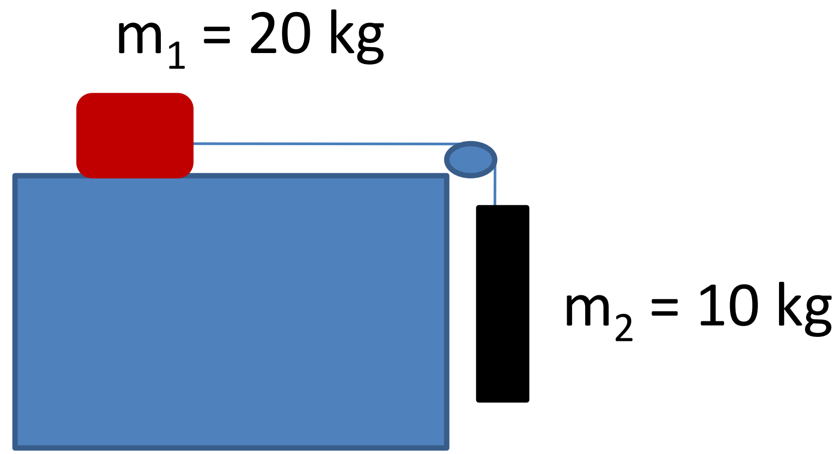
$$T - W = -ma$$

$$T = W - ma$$

$$T = mg - ma = m(g - a)$$

Example:

Mass 1 is on frictionless surface. Mass 2 is allowed to fall. What accelerations do the objects have. (They are tied to together.)



Free Body Diagrams

Do Newton's 2nd Law for each mass

Mass 1 (moves horizontally, picking right for positive)

$T = m_1 a_1$

Mass 2 (moves vertically, picking up for positive)

$T - m_2 g = - m_2 a_2$

Important: $a_1 = a_2$ in magnitude, since they are tied together.

$T = m_1 a$ and $T = m_2 g - m_2 a$

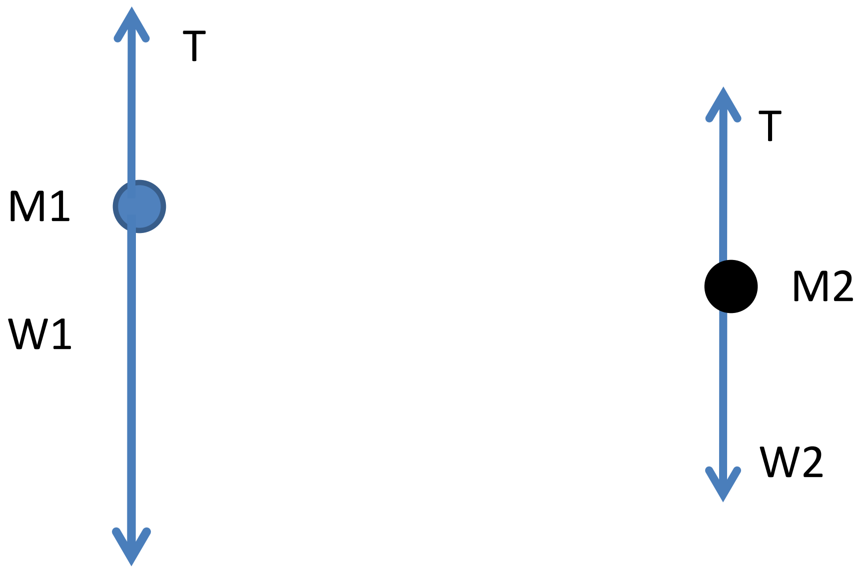
$m_1 a = m_2 g - m_2 a$

$$a = \frac{m_2 g}{m_1 + m_2}$$

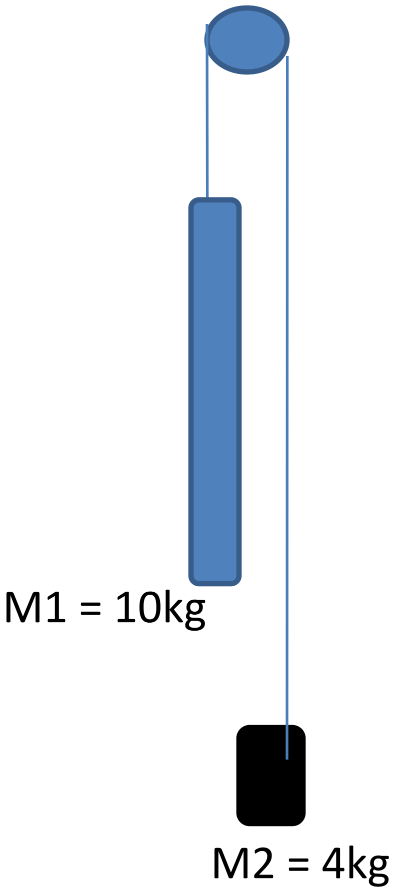
Problems

How long does it take for the heavier block to fall 2 meters?

Free Body Diagrams



See ex 4.10 Atwood Machine.



Newton's 2nd Law for each body:

Mass1

$F = ma$

$T - m_1 g = - m_1 a_1$

Mass 2

$T - m_2 g = m_2 a_2$ (masses accelerate together a1 = a2)

$T = m_1 g - m_1 a$ and $T = m_2 g + m_2 a$

So: $m_1(g - a) = m_2(g + a)$

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$$

Looking at this problem from another perspective.

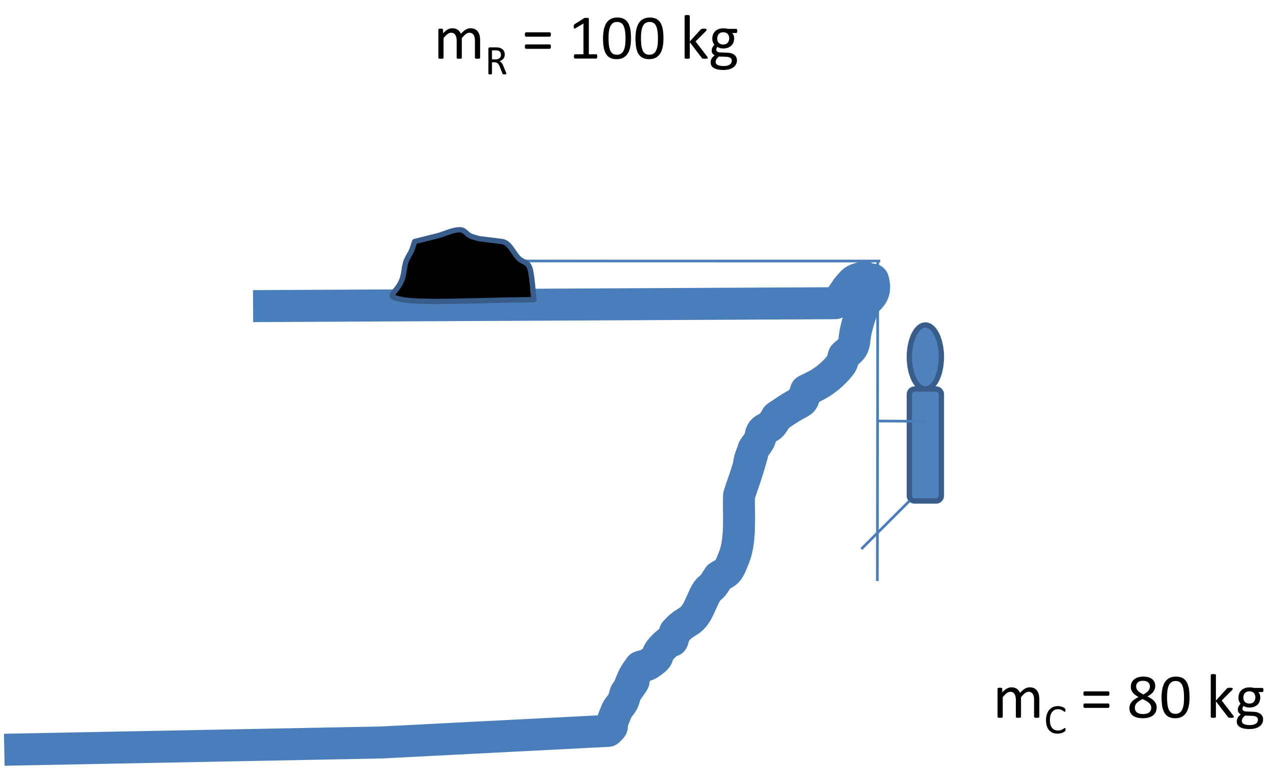
m_1g is pulling one way.
m_2g is pulling the other way.

Their sum is acting on the total mass $(m_1 + m_2)$

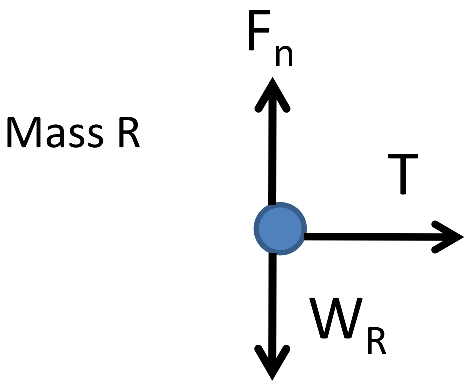
$\Sigma F = ma$
$m_1g - m_2g = (m_1 + m_2)a$

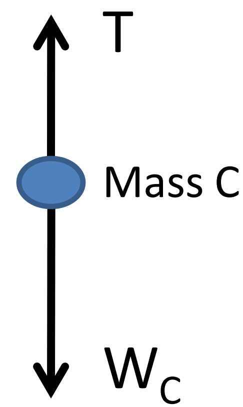
$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$$

A 80 kg climber is dangling over a cliff via a rope tied to a 100 kg rock. If the rock is 10 meters from the edge, how long before the rock reaches the edge? Assume no friction between the rock and the ground.



First find the acceleration of the climber and the rock.





Newton 2nd Law (F = ma)

Rock: $T = m_R a$

Climber: $T - W_C = -m_C a$

$T = W_C - m_C a$

$$a = \frac{m_C g}{m_C + m_R} = \frac{(80\,kg)g}{80\,kg + 100\,kg} = 4.4\,m/s^2$$

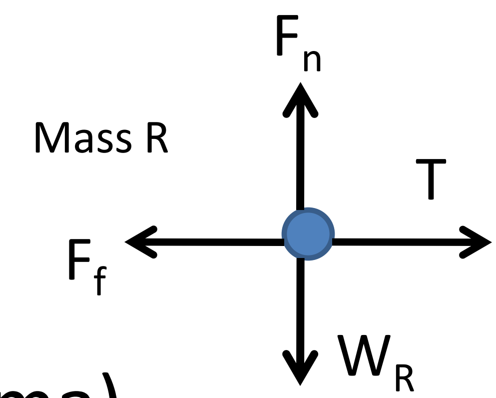
Now that we know the rock is accelerating at 4.4 m/s^2, we can calculate the time before the rock reaches the edge.

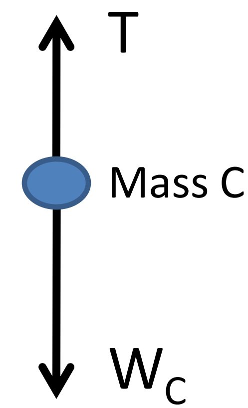
Use: $\Delta x = v_0 t + \frac{1}{2} a t^2$

$10m = \frac{1}{2} (4.4 m/s^2) t^2$

$10m = 2.1\ s$

Repeat previous problem but with friction. Let the coefficient of friction between the rock and the ground be $\mu_k = 0.7$.





Newton 2nd Law (F = ma)

Rock: $T - F_f = m_R a$

$T = m_R a + \mu_s F_n$

$T = m_R a + \mu_s m_R g$

Climber: $T - W_C = -m_C a$

$T = W_C - m_C a$

So: $m_R a + \mu_k m_R g = m_C g - m_C a$

- $m_R a + \mu_k m_R g = m_C g - m_C a$
- Solve for a gives:

$$a = \frac{m_C g - \mu_k m_R g}{m_C + m_R} = \frac{(80\,kg)g - 0.7(100\,kg)g}{80\,kg + 100\,kg} = 0.54\,m/s^2$$

Since the acceleration is now only $0.54 m/s^2$, it takes longer for the rock to reach the edge.

$\Delta x = v_0 t + ½ at^2$

$10m = ½ (0.54m/s^2)t^2$

10m = 6.1 s

What happens if the coefficient of friction between the rock and the ground is 0.81?

The force that want to pull the climber down and tugs on the rock is still the weight of the climber, $m_C g$ = (80kg)g = 784 N

If μ_s = 0.81, the static frictional force that is needed to be overcome to begin sliding the rock is:

$0.81F_n = 0.81m_R g = 0.81(100kg)g = 793.8$ N

The rock will not slide!

A 70 kg person jumps off a 5 meter ledge. Once the person comes in contact with the ground, it takes 0.8 seconds to come to a complete stop. Find the magnitude of the force, the ground exerts on the person during this time.

First find the speed that the person hits the ground.

Use: $v_f^2 = v_0^2 + 2a\Delta x = 0^2 + 2g5m$

$v_f = 9.9$ m/s

Person is falling 9.9 m/s when he hits the ground. Now over the next 0.8 seconds, he comes to a stop.

Use: $v_f = v_0 + a\,t$ to solve for acceleration.

$0\ m/s = 9.9\ m/s + a(0.8\ s)$

$a = -12.4\ m/s^2$

Find the magnitude of the force the ground exerts.

Use: $F = ma$

$F = (70\ kg)\ 12.4 m/s^2 = 868\ N$