

# **Physics 2018: Great Ideas in Science: The Fall 2008 Physics Module**

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**Edition 2.0**

## **Abstract**

These class notes are designed for use of the instructor and students of the course **Physics 2018: Great Ideas in Science**. This edition was last modified for the Fall 2008 semester.

### III. Cosmology — The Study of the Origin, Structure, and Evolution of the Universe

#### A. Hubble's Observations and Hubble's Law.

1. Stars are thermonuclear furnaces that emit their own light.
  - a) Planets on the other hand do not emit their own light, they just reflect a star's light.
  - b) Stars are gravitationally clumped together in bigger collections known as **galaxies**.
  - c) The **Milky Way**, our home galaxy, contains over 200 billion stars!
  
2. In 1931, Edwin Hubble and Milton Humason published a paper that showed that the fainter a galaxy appeared, the higher its spectrum was redshifted  $\implies$  **Hubble's Law**.

- a) It was proposed that the redshift resulted from the **Doppler Effect**:

$$z = \frac{\Delta\lambda}{\lambda_o} = \frac{v_r}{c}, \quad (\text{III-1})$$

where  $z$  is the redshift,  $\Delta\lambda = \lambda - \lambda_o$  is the spectral line wavelength shift,  $\lambda$  is the observed wavelength,  $\lambda_o$  is the *rest* (or laboratory) wavelength,  $v_r$  is the radial (*i.e.*, line-of-sight) velocity, and  $c$  is the speed of light.

- b) Note that Eq. (III-1) is valid only if the velocity of the galaxy is small with respect to the speed of light (*i.e.*,  $v_r \ll c$ ). If  $v_r \lesssim c$ , then the *relativistic* form of the Doppler Effect must be used:

$$z = \frac{\Delta\lambda}{\lambda_o} = \frac{\sqrt{1 + v_r/c}}{\sqrt{1 - v_r/c}} - 1. \quad (\text{III-2})$$

- c) Rewriting this relativistic formula, we can express velocity as a function of redshift:

$$\frac{v_r}{c} = \frac{(z + 1)^2 - 1}{(z + 1)^2 + 1} . \quad (\text{III-3})$$

You will note that there is no way for a galaxy's velocity to exceed that of light when using the relativistic form of the Doppler Effect.

- d) They also proposed that the fainter a galaxy appeared, the farther away it was. Hubble and Humason came up with the first extragalactic standard brightness markers to estimate distance  $\implies$  **distance indicators**.
- e) Hence, the more distant a galaxy (*i.e.*, fainter galaxies), the larger the redshift or recession velocity

$\implies$  **The Universe is Expanding.**

- f) Hubble's Law mathematically:

$$v_r = H_o d , \quad (\text{III-4})$$

where  $v_r$  is the recession velocity,  $d$  is the distance to the galaxy, and  $H_o$  is Hubble's constant.

3. Prior to the launch of the Hubble Space Telescope (HST),  $H_o$  was not accurately known  $\implies$  values ranged between 50 to 90 km/sec/Mpc ('Mpc' = mega-parsec, or million  $[10^6]$  parsecs and 1 parsec = 3.26 light years) depending on the technique used to measure it.
4. The primary reason the HST was built was to determine an accurate value for  $H_o$ . This is one of the so-called *Key Projects* of HST.

5. This HST Key Project has ascertained the following value for Hubble's constant (see Freedman *et al.* 2001, *Astrophysical Journal*, **553**, 47):

$$H_o = 72 \pm 8 \text{ km/sec/Mpc} \quad (\text{HST Result}). \quad (\text{III-5})$$

6. The WMAP mission to map the cosmic microwave background (see below) has determined an even higher precision to Hubble's constant:

$$H_o = 71 \pm 4 \text{ km/sec/Mpc} \quad (\text{WMAP Result}) \quad (\text{III-6})$$

(see the WMAP website at <http://map.gsfc.nasa.gov/>).

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**Example III-1.** If we see an Fe line at 4800 Å, which at rest is at 4000 Å, how far away is the galaxy?

$$\begin{aligned} v_r &= \frac{\Delta\lambda}{\lambda_o} c = \frac{4800 \text{ \AA} - 4000 \text{ \AA}}{4000 \text{ \AA}} (3.00 \times 10^5 \text{ km/s}) \\ &= \frac{800 \text{ \AA}}{4000 \text{ \AA}} (3.00 \times 10^5 \text{ km/s}) \\ &= 0.2 (3.00 \times 10^5 \text{ km/s}) = 6.00 \times 10^4 \text{ km/s} \\ d &= \frac{v_r}{H_o} = \frac{6.00 \times 10^4 \text{ km/s}}{71 \text{ km/s/Mpc}} = 8.49 \times 10^2 \text{ Mpc} \end{aligned}$$

$$\boxed{d = 850 \text{ Mpc.}}$$

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7. Velocity in Hubble's Law represents the expansion velocity of the Universe as a whole. This Hubble velocity is referred to as the **Hubble Flow**.

8. A galaxy's recession velocity will not exactly match the velocity as predicted by Hubble's Law since this law is based on the velocity distribution as a function of distance of numerous galaxies. Any excess velocity that a galaxy retains when the Hubble Flow is subtracted out is called a galaxy's **peculiar velocity**.

## B. Olber's Paradox — Why does it get dark at night?

1. During the 19th and beginnings of the 20th centuries, the scientific viewpoint was that the Universe is unchanging (*static*), has no spatial boundaries (*infinite*), and had no beginning (*eternal*). If the Universe is static, infinite, and eternal, we should light in every direction we look.
  - a) This statement is referred to as **Olber's Paradox**.
  - b) Actually, Halley (of the comet fame), was the first to pose this question, Olber was the first to formally publish it in 1826.
  - c) The inverse-square law of light cannot be used as a solution to Olber's Paradox for the following reasons:
    - i) The luminosity (*i.e.*, brightness) at the surface of a star is given by  $L = 4\pi R^2 F$ , where  $R$  is the radius of the star and  $F$  is the energy flux (energy per unit time) of the star at its surface.
    - ii) The flux of a star falls off as  $r^{-2}$ , where  $r$  is the distance to the star:  $f = (R/r)^2 F$ , where  $f$  is the *observed* flux.
    - iii) For a uniform distribution of stars, each successive shell radially away from the Earth will contain  $4\pi r^2 N$  stars, where  $N$  is the areal density [ $\text{cm}^2$ ] of stars on a shell at surface area  $4\pi r^2$ .

- iv) The total flux that should arrive at Earth from all of the stars in the Universe is a convolution of these two quantity giving  $f_{\text{obs-shell}} = (4\pi r^2 N)(f) = (4\pi r^2 N)(R/r)^2 F = LN \implies$  the sky should be ablaze with light for an infinitely large eternal Universe!
  - v) Even for a non-uniform distribution of stars, the inverse-square law will fail to account for the dark night sky for an infinite, eternal Universe.
- d) Though not known when Olber's Paradox was published, interstellar absorption also cannot be used to explain the dark night sky.
- i) Absorption from gas and dust in the interstellar medium (ISM) and the intergalactic medium (IGM) would heat the gas and dust.
  - ii) Over an infinite amount of time, this dust and gas would reach an equilibrium temperature that is equal to the radiative temperature of the integrated light from the stars.
  - iii) Hence, the ISM and IGM would shine as bright as the surface of the stars contained in the Universe.
2. The solution to Olber's Paradox is that the Universe is expanding (hence not static) and is not eternal  $\implies$  **Big Bang Theory** — it had a beginning!
- a) Light gets redshifted out of the visible band for stars and galaxies at large distances.

- b) As we look out, we look back in time. We cannot look infinitely far out since, sooner or later, we will see the Big Bang.
3. How long ago did this happen? We will treat this question in a very simplified way initially: Calculate when all the galaxies were at the same position from Hubble's Law.
- a) Since there is still some uncertainty to the value of Hubble's constant, we will use the following scale factor formalization:

$$\begin{aligned} H_0 &= 100 h \text{ km/s/Mpc} = 3.24 \times 10^{-18} h \text{ s}^{-1} \\ &= 1.02 \times 10^{-10} h \text{ yr}^{-1}, \end{aligned} \quad (\text{III-7})$$

where  $h$  (the Hubble constant scale factor) = 1.0 if  $H_0 = 100 \text{ km/s/Mpc}$  and  $h = 0.5$  if  $H_0 = 50 \text{ km/s/Mpc}$ .

- b) From Newtonian mechanics, the distance to a galaxy is  $d = v_r t$ , where  $v_r$  is the galaxy's radial velocity (assumed constant here), and  $t$  is the time since the galaxy started at the origin. But from Hubble's Law,  $v_r = H_0 d$ , so

$$t_H = \frac{d}{v_r} = \frac{d}{H_0 d} = \frac{1}{H_0}$$

or

$$\boxed{t_H = 9.78 \times 10^9 h^{-1} \text{ yr} .} \quad (\text{III-8})$$

- c) The Universe actually younger than that due to gravity slowing down the expansion over time. Hence the time given in Eq. (III-8) corresponds to the maximum age of the Universe which is referred to as the **Hubble Time**.
- i) If  $h = 1.0$  ( $H_0 = 100 \text{ km/s/Mpc}$ ), then  $t_H = 9.78$  billion years.

- ii) If  $h = 0.5$  ( $H_o = 50$  km/s/Mpc), then  $t_H = 19.6$  billion years.
- iii) The best value for Hubble's constant (as determined by HST and WMAP, see Eqs. III-5 and III-6) is  $h = 0.71$  ( $H_o = 71$  km/s/Mpc), and  $t_H = 13.7$  billion years.
- d) If a galaxy is farther than  $d_{\text{obs}} = ct_H$  ( $= 9.78, 13.7,$  or  $19.6$  billion light years, depending upon the value of  $H_o$ ) away, we will never see it since light would not have had enough time to reach us  $\implies d_{\text{obs}}$  is the size of the **observable universe**.

## C. The Big Bang Theory

1. The Universe started in an extremely small, hot, and dense state. As we go backwards in time, the Universe gets progressively smaller, hotter, and denser.
  - a) The Big Bang occurred everywhere in space, not just at one location  $\implies$  we are in the Big Bang!
  - b) Galaxies were not thrown apart  $\implies$  the fabric of space itself is expanding and the galaxies move apart as a result.
2. The **Cosmological Principle** — assumptions of the Big Bang.
  - a) **Homogeneity** — matter is uniformly distributed in space on a very large scale ( $d > 100$  Mpc).
  - b) **Isotropy** — the Universe looks the same in every direction.
  - c) **Universality** — physical laws are the same everywhere in the Universe at all times.

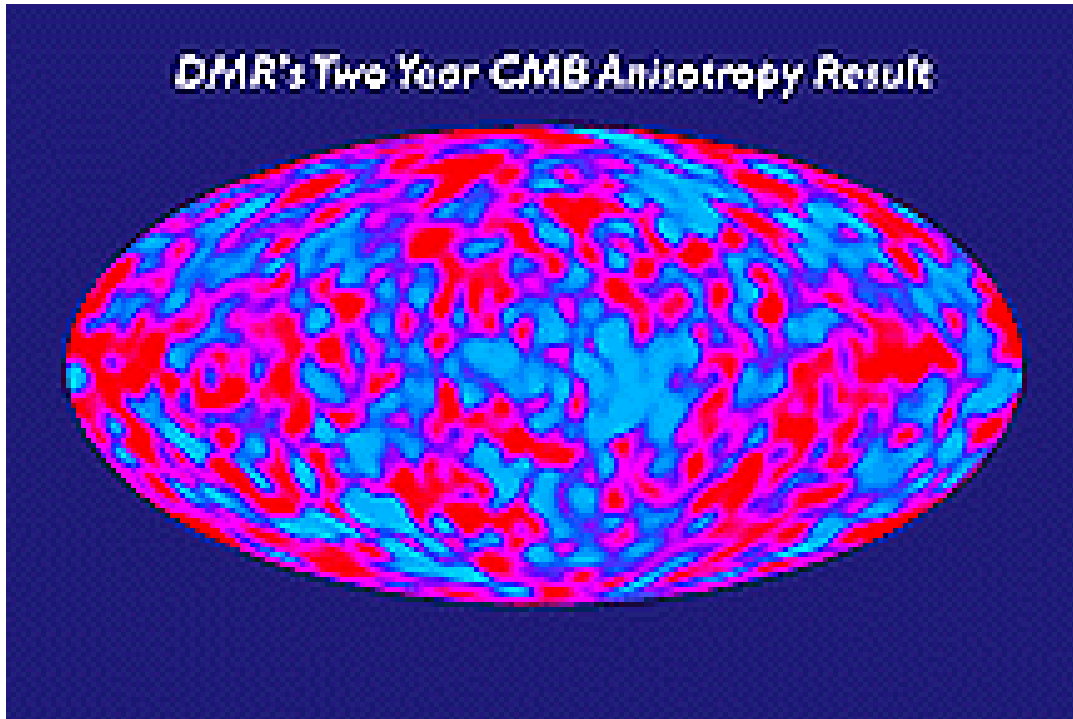


Figure III-1: Cosmic Microwave Background as observed by COBE. This microwave image of the sky shows the variations in the Universe's temperature at recombination time. Light grey are regions of cooler gas, dark grey are regions of hotter gas.

- d) **Cosmological Redshifts** — redshifts are caused by the expansion of the Universe through the Doppler Effect.
3. We see the Big Bang fireball in every direction as microwave blackbody radiation  $\implies$

**3 K Cosmic Microwave Background (CMB) radiation.**

- a) When this light was emitted, the Universe was very hot.
- b) As the Universe expanded, this light was redshifted until today it is microwave light ( $z = 1000!$ , the farthest quasar is at  $z = 4.9$ ).
- c) Penzias and Wilson discovered this CMB in the early 1960's, confirming theoretical predictions of the Big Bang Theory made by Dicke and Peebles. Penzias and Wilson

later won a Nobel Prize for their discovery.

- d)** The COsmic Background Explorer (COBE) spacecraft was launched in the early 1990's to investigate this background radiation (see Figure III-1).
    - i)** Found the Universe radiates as a perfect black-body (after the solar system's motion is subtracted) at a temp of 2.73 K.
    - ii)** Small variations in the thermal distribution in the CMB on the order of 1 part in 100,000 (*i.e.*, the *intrinsic anisotropy*) show that by the time this radiation was emitted (380,000 years after the Big Bang), inhomogeneities in the mass-energy of the Universe had begun which would later form the galaxies.
  - e)** More recently, the Wilkinson Microwave Anisotropy Probe (WMAP) has observed this background at even higher spatial resolution.
    - i)** Has determined the most accurate value of Hubble's constant (71 km/sec/Mpc).
    - ii)** The age of the Universe (*i.e.*, the time since the Big Bang) is 13.7 billion ( $1.37 \times 10^{10}$ ) years.
    - iii)** We live in a flat (in a 3-dimensional sense) space-time (discussed in detail shortly).
- 4.** As the Universe expands, it should slow down due to the gravitational pull of one galaxy on another.

a) Are there enough galaxies (*i.e.*, mass) to stop the expansion?

i) The current critical density is given by the expression

$$\rho_c = \frac{3H_o^2}{8\pi G} = 1.88 \times 10^{-29} h^2 \text{ gm/cm}^3, \quad (\text{III-9})$$

which ranges from  $4.70 \times 10^{-30} \text{ gm/cm}^3$  for  $H_o = 50 \text{ km/s/Mpc}$  to  $1.88 \times 10^{-29} \text{ gm/cm}^3$  for  $H_o = 100 \text{ km/s/Mpc}$ . For the WMAP value for  $H_o$ , the current value of the critical density is  $9.48 \times 10^{-30} \text{ gm/cm}^3$ .

ii) If the Universe's mass density,  $\rho$ , is less than a critical density,  $\rho < \rho_c$ , gravity will not halt the expansion  $\implies$  an **Open Universe**.

iii) If  $\rho > \rho_c$ , gravity will halt the expansion and cause a contraction down to a *Big Crunch*!  $\implies$  a **Closed Universe**.

iv) The Universe may be able to rebound (Big Bang) and start over again  $\implies$  an **Oscillating Universe**.

v) If  $\rho = \rho_c$ , gravity will halt the expansion after an infinite amount of time  $\implies$  a **Flat Universe**.

b) The deceleration of the Universe is measured with the quantity  $q_o$  — the **deceleration parameter**.

$$q_o = \frac{8\pi G}{3} \frac{\rho_o}{H_o^2}, \quad (\text{III-10})$$

where  $\rho_o$  is the current mass-energy density of the Universe.

- i) For an open universe,  $q_o < 1/2$ .
  - ii) For a closed universe,  $q_o > 1/2$ .
  - iii) For a flat universe,  $q_o = 1/2$ .
  - iv) If there were no mass in the Universe,  $q_o = 0$ .
- c) Another parameter that is often used is the density parameter:  $\Omega_o = \rho_o/\rho_c$ . By knowing the value of  $\Omega_o$ , we will know the overall shape (*i.e.*, **curvature**) of the Universe in 4 dimensions and know the final fate of the Universe (see Figure III-2):

### Structure of the Universe

Type	Geometry	Curvature	Density Parameter	Deceleration Parameter	Age
Closed	Spherical	Positive	$\Omega_o > 1$	$q_o > \frac{1}{2}$	$T_o < \frac{2}{3}(1/H_o)$
Flat	Flat	Zero	$\Omega_o = 1$	$q_o = \frac{1}{2}$	$T_o = \frac{2}{3}(1/H_o)$
Open	Hyperbolic	Negative	$0 < \Omega_o < 1$	$0 < q_o < \frac{1}{2}$	$\frac{2}{3}(1/H_o) < T_o < 1/H_o$
No Matter	Hyperbolic	Negative	$\Omega_o = 0$	$q_o = 0$	$T_o = 1/H_o$

- d) What is  $\rho$  of the Universe?
- i) Galaxy counting:  $\rho_{gal} = 3 \times 10^{-28} \text{ kg/m}^3$   
(25 times too small  $\rightarrow$  open)
  - ii) Deuterium ( $^2\text{H}$ ) abundance (baryons):  $\rho_b = \rho_{matter} = 7 \times 10^{-28} \text{ kg/m}^3$   
( $\rho_{matter}$  is 11 times too small  $\rightarrow$  open)
  - iii) Light:  $\rho_{rad} = a T_{rad}^4/c^2$  and  $T_{rad} = 2.73 \text{ K}$ ,  

$$\rho_{rad} = 6.5 \times 10^{-31} \text{ kg/m}^3$$

$$\rho_{rad} \ll \rho_{matter}$$
    - Today *matter* dominates the Universe!
    - Radiation dominated at earlier times.

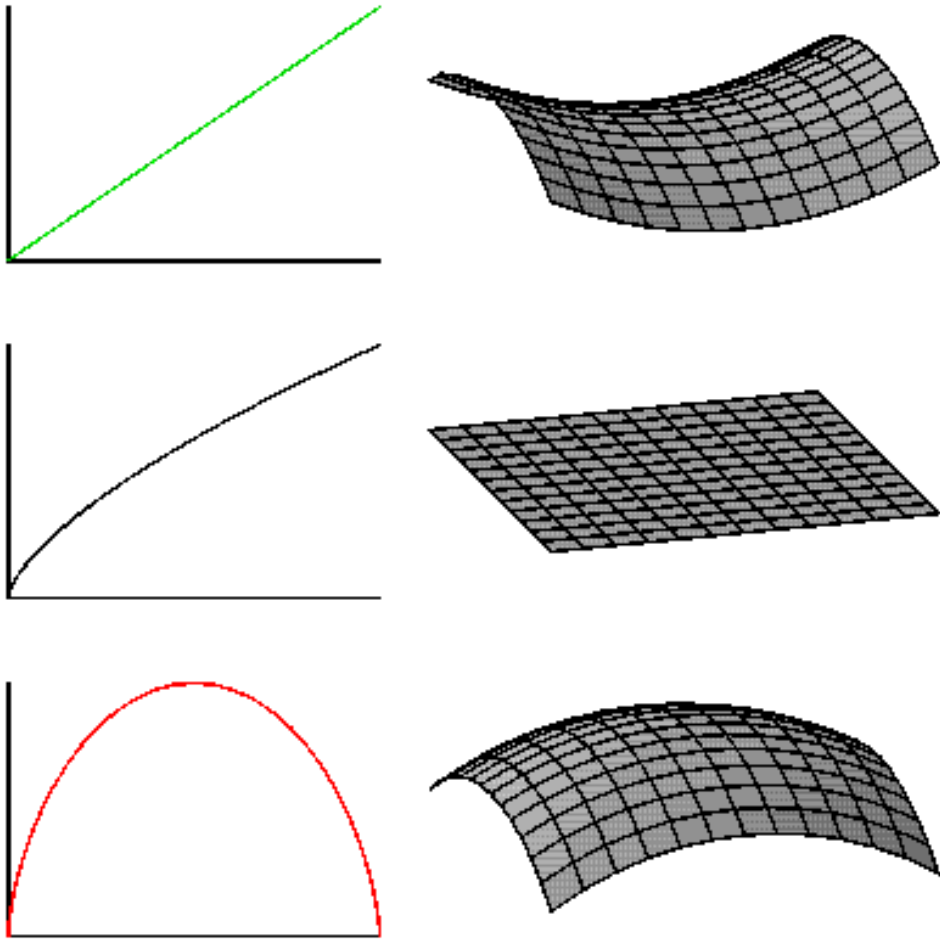


Figure III-2: Three histories and structures of the Universe with respect to  $\Omega_o$ . The *top* panel shows a universe with  $\Omega_o < 1$  ( $\rho < \rho_c$ ). Such a universe will continue to expand forever as shown in the plot where size of the universe  $R$  is plotted as a function of time  $t$ . Also, this universe has a *negative* curvature — a hyperbolic (*i.e.*, saddle) shape. The *middle* panel shows a universe with  $\Omega_o = 1$  ( $\rho = \rho_c$ ) which is flat (*zero* curvature) and gravity will just halt the expansion after an infinite amount of time. The *bottom* panel shows a *positive* curvature (*i.e.*, spherical shape) with  $\Omega_o > 1$  ( $\rho > \rho_c$ ). This type of universe will grow to a maximum size, then start to contract to a Big Crunch.

5. If the Universe had gone through an *Inflationary stage* (see below), then the Universe is essentially flat and  $q_o = 1/2$ .

a) One of the key projects of the *Hubble Space Telescope* is to accurately measure Hubble's constant ( $H_o$ ). This project has determined that  $H_o = 72$  km/s/Mpc with an uncertainty of 10% (note that WMAP measures it more accurately at 71 km/s/Mpc with an uncertainty of 4%).

b) Using these values of  $q_o$  and  $H_o$ , we can ascertain the current mass-energy density of the Universe:

$$\begin{aligned}\rho_o &= \frac{3H_o^2 q_o}{8\pi G} = \frac{3 \cdot \left( \frac{70 \text{ km/s} \times 10^3 \text{ m/km}}{10^6 \text{ pc} \times 3.0856 \times 10^{16} \text{ m/pc}} \right)^2 \cdot (1/2)}{8\pi (6.668 \times 10^{-11} \text{ N m}^2/\text{kg}^2)} \\ &= 4.6 \times 10^{-27} \text{ kg/m}^3 ,\end{aligned}$$

or 6.6 times bigger than the baryonic matter we can measure (and see).

c) As such, astronomers have introduced a term called **dark matter** — matter which is not made up of protons and neutrons and cannot easily be seen.

d) So, 87% of the Universe seems to be composed of this dark matter! The race is on throughout the astrophysical community to try and find the identity of this hidden mass.

e) Also note that galactic rotation curves also show that there is a large amount of matter in the vicinity of galaxies that cannot be seen.

6. Recently, distance-redshift measurements of supernovae explosions in distant galaxies indicate, however, that the Universe may be accelerating instead of decelerating!

- a) In analogy with “dark matter,” astronomers refer to the cause of this acceleration as **dark energy**. Recent measurements based on the brightness of Type Ia supernovae suggest that dark energy accounts for 73% of the total Universe’s mass-energy, while dark matter accounts for 23%, and finally normal matter accounts for only 4% of this total mass-energy.
- b) What might this dark energy be? The field equations from general relativity produce a constant term called the **cosmological constant** ( $\Lambda$ ).

- i) Einstein’s field equations for the Universe produce an *equation of motion* (like Newton’s 2nd law) of

$$a(t) = (-4\pi G\rho(t) - \Lambda) \frac{R(t)}{3} , \quad (\text{III-11})$$

where  $a$  is the acceleration of the Universe’s expansion,  $\rho$  is the mass-energy density,  $R$  is the *size* of the Universe, and  $t$  is the time since the Big Bang.

- ii) As can be seen,  $a$ ,  $\rho$ , and  $R$  are all functions of time — they can change over time.
- iii) If the Universe is *slowing up*, then its acceleration is less than zero ( $a < 0$ ) — a deceleration.
- iv) If the Universe is *speeding up*, then its acceleration is greater than zero ( $a > 0$ ).
- v) If we lived in a static Universe (one that doesn’t change with time), the cosmological constant would have to equal  $-4\pi G\rho$ , since ‘ $a$ ’ would have to be zero in such a case, due to the fact that  $\Delta v$  and  $\Delta R$

would both have to be zero over time. From this, the mass-energy density would remain constant for all time (since  $\Lambda$  is constant and the Universe's size would not be changing).

**vi)** From this realization, the cosmological constant works in the opposite sense as gravity (*i.e.*,  $G\rho$ ) due to the negative sign  $\implies$  **it acts like an anti-gravity** (assuming  $\Lambda$  has a positive value).

**vii)** The term  $\Lambda R/3$  is sometimes called the **cosmic repulsion term** of the Universe's equation of motion.

- c)** Einstein originally had a non-zero value for this to keep the Universe from changing size  $\implies$  astronomers and physicists of the late 19th and early 20th centuries thought that the Universe was *static*.
- d)** When Einstein learned from Hubble that the Universe was expanding, he went back and set  $\Lambda = 0$  to let his model for the Universe expand.
- e)** A non-zero value for  $\Lambda$  allows for the possibility that empty space-time ( $\rho = 0$ , a universe with no matter in it) might be curved.
- f)** Note that the actual value for  $\Lambda$  is set from the initial conditions of the Universe at time zero. Since we can't see past the CMB, we cannot see the Big Bang itself. Hence, we have to try and determine this cosmological constant from observations coupled with the field equations of general relativity.

- g) Although it would require a little calculus (which we won't show here), under a few assumptions of the geometry of the Universe, an expression for the cosmological constant can be expressed in terms of current values of Hubble's constant ( $H_0$ ), the deceleration parameter ( $q_0$ ), and mass-energy density of the Universe ( $\rho_0$ ):

$$\Lambda = 4\pi G\rho_0 - 3q_0H_0^2 \quad . \quad (\text{III-12})$$

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**Example III-2.** What must the cosmological constant be for the best estimates of the parameters listed in Eq. (III-12)?

$$\begin{aligned} \Lambda &= 4\pi(6.668 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(4.6 \times 10^{-27} \text{ kg/m}^3) - \\ &\quad 3\left(\frac{1}{2}\right) \left(\frac{70 \text{ km/s} \times 10^3 \text{ m/km}}{10^6 \text{ pc} \times 3.0856 \times 10^{16} \text{ m/pc}}\right)^2 \\ &= -3.9 \times 10^{-36} \text{ s}^{-2} \quad . \end{aligned}$$

From this, we can calculate what the acceleration of the Universe is due to this cosmological constant term (use the best estimate of the radius of the Universe to be 13.7 billion light years =  $1.3 \times 10^{26}$  m):

$$\begin{aligned} a_\Lambda &= \frac{\Lambda R}{3} = \frac{(-3.9 \times 10^{-36} \text{ s}^{-2}) \cdot (1.3 \times 10^{26} \text{ m})}{3} \\ &= -1.7 \times 10^{-10} \text{ m/s}^2 \quad . \end{aligned}$$

Now, since the acceleration due to gravity on the Earth's surface is  $9.80 \text{ m/s}^2$ , that's 54 billion times larger than the acceleration due to the cosmological constant — unmeasurable on the Earth's surface (or by any large gravitating body). It only shows its presence on the very large scale!

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7. Making use of the physics that Einstein explained in the General Theory of Relativity in conjunction with the cosmological principle, Alexander Friedmann developed a set of equations in 1922 that described the expansion of the Universe:

$$\dot{R}^2(t) = -kc^2 + [8\pi G \rho(t) + \Lambda] \frac{R^2(t)}{3}, \quad (\text{III-13})$$

where  $k$  is the curvature index (+1 for *positive* curvature, 0 for *flat* curvature, and -1 for *negative* curvature),  $\rho$  is the mass-energy density at time  $t$ ,  $c$  is the speed of light,  $R$  is the scale factor (*related to the size*) of the Universe at time  $t$ , and  $\Lambda$  is the cosmological constant.

- a) By setting  $\Lambda = 0$ , we can rewrite Eq. (III-13) as

$$\left[ \left( \frac{1}{R} \frac{dR}{dt} \right)^2 - \frac{8\pi}{3} G \rho \right] R^2 = -kc^2, \quad (\text{III-14})$$

which is known as the **Friedmann equation**. Figure III-3 shows graphical solutions to Eq. (III-14) for evolutionary models of the 3 types of geometries for the Universe.

- b) With the cosmological constant, Friedmann's equation becomes

$$\left[ \left( \frac{1}{R} \frac{dR}{dt} \right)^2 - \frac{8\pi}{3} G \rho - \frac{1}{3} \Lambda \right] R^2 = -kc^2, \quad (\text{III-15})$$

which I shall refer to as the *modified* Friedmann equation. Figure III-4 shows graphical solutions to Eq. (III-15) for evolutionary models of a positive, zero, and negative value for the cosmological constant in flat space.

## D. Particles and Forces

1. Before discussing the history of the Universe, we need to understand the 4 natural forces in nature and their effects on particles in the Universe.

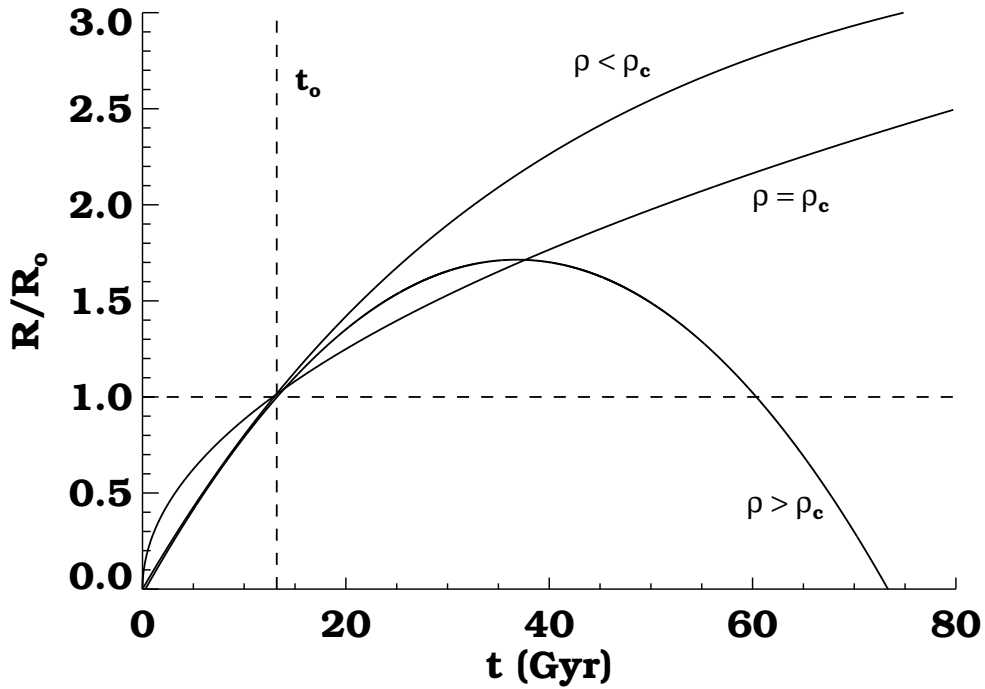


Figure III-3: Histories of Friedmann model universes ( $\Lambda = 0$ ). Note that the present size of the Universe (hence, present time) is indicated by  $R = 1$ .

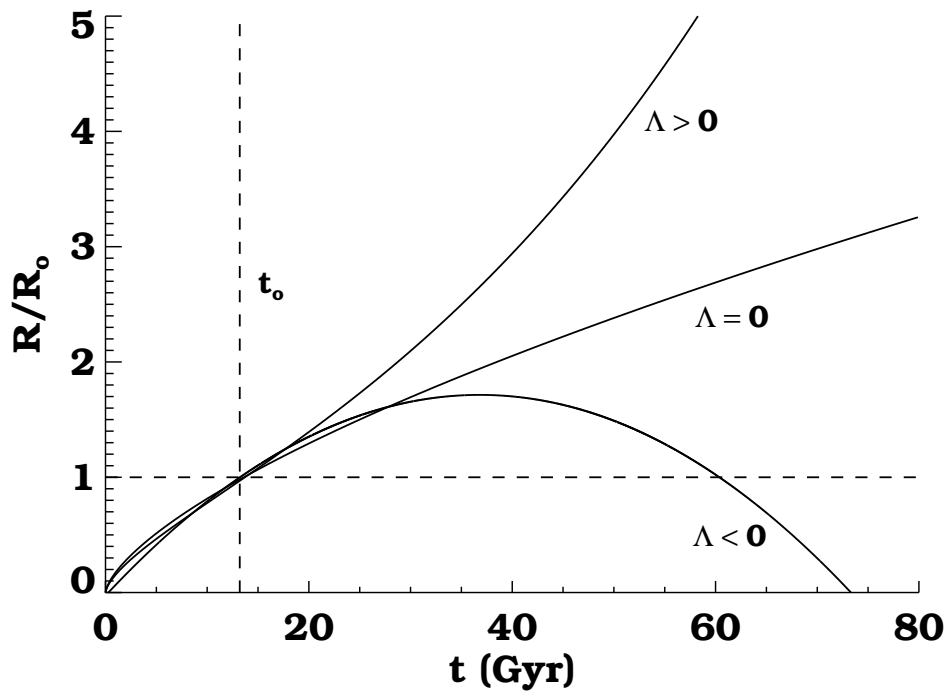
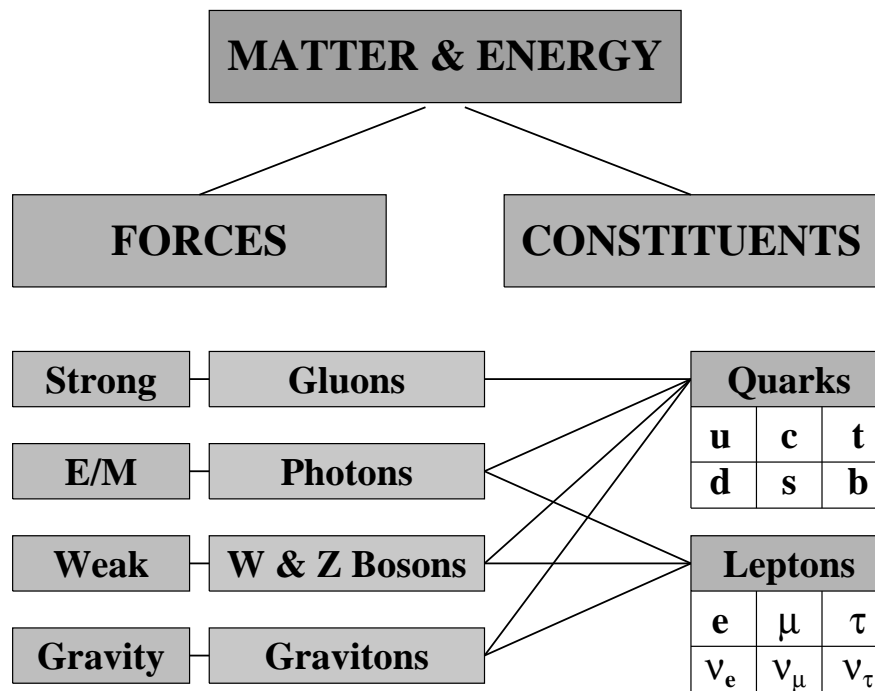


Figure III-4: Histories of flat ( $k = 0$ ) modified Friedmann model universes. Note that the present size of the Universe (hence, present time) is indicated by  $R = 1$ .

2. There are 4 natural forces (*i.e.*, those forces associated with force fields). In order of strength they are:
  - a) **Strong interactions:** Force that binds nucleons together — acts over a range of  $\sim 10^{-13}$  cm. **Hadrons** participate in the strong force. The smallest component particle of a hadron is called a **quark**. This force is mediated by field particles called **gluons**.
  - b) **E/M interactions:** Force between charged particles which has an infinite range that falls off as  $1/r^2$ . This force is 100 times weaker than the strong force, however it is what holds atoms and molecules together. This force is mediated by the **photon** field particle.
  - c) **Weak interactions:** These are responsible for  $\beta$ -decay of nuclei (*i.e.*, radioactivity) —  $10^{-13}$  times as strong as strong interactions with a range  $\ll 10^{-13}$  cm. The **intermediate vector boson** (often called **weakons**) mediates this force.
  - d) **Gravitational interactions:** These are by far the weakest of the interactions on the microscopic scale, typically about  $10^{-40}$  times as strong as the strong interactions on nuclear scales. Gravity is another infinite,  $1/r^2$  force, except it is charge independent — as such, this force dominates all others on a cosmic scale. The (yet to be discovered) **graviton** has been proposed as the particle that mediates the gravitational force.
3. There are 2 main groups of particles that make up all matter and energy:
  - a) **Elementary particles:** These are particles that make up matter. They are subdivided into 3 groups:

- i) **Leptons** (*light* particles) include the *electron* ( $e^-$ ,  $m_e = 511$  keV,  $1$  keV =  $1000$  eV,  $1$  eV =  $1.60 \times 10^{-19}$  Joules), *muon* ( $\mu$ ,  $m_\mu = 107$  MeV), and *tau particle* ( $\tau$ ,  $m_\tau = 1784$  MeV), each with a negative charge; their respective neutrinos: *electron neutrino* ( $\nu_e$ ,  $m_{\nu-e} < 30$  eV), *muon neutrino* ( $\nu_\mu$ ,  $m_{\nu-\mu} < 0.5$  MeV), and *tau neutrino* ( $\nu_\tau$ ,  $m_{\nu-\tau} < 250$  MeV), each with no charge; and the antiparticles of each:  $e^+$  (called a *positron*),  $\bar{\mu}$ ,  $\bar{\tau}$ ,  $\bar{\nu}_e$ ,  $\bar{\nu}_\mu$ , and  $\bar{\nu}_\tau$ . These particles do **not** participate in the strong interactions. All leptons have spin of  $1/2$ .
- ii) **Mesons** are particles of intermediate mass that are made of quark-antiquark pairs and include *pi-ions*, *kaons*, and  *$\eta$ -particles*. All are unstable and decay via weak or E/M interactions. All mesons have either 0 or integer spin.
- iii) **Baryons** (*heavy* particles) include the nucleons  $n$  (*neutrons* — neutral particles) and  $p$  (*protons* — positive charged) and the more massive *hyperons* (*i.e.*,  $\Lambda$ ,  $\Sigma$ ,  $\Xi$ , and  $\Omega$ ). Baryons are composed of a triplet of quarks. Each baryon has an antibaryon associated with it and has a spin of either  $1/2$  or  $3/2$ .
- b) **Field particles:** These particles mediate the 4 natural forces as mentioned above: **gluons**, **photons**, **weakons**, and **gravitons**. These are the *energy* particles.

4. From the above list of elementary particles, there seems to be only 2 types of basic particles: *leptons* which do not obey the strong force and *quarks* which do obey the strong force. There are 6 *flavors* of leptons (as describe above). As such, it was theorized and later observed, 6 flavors or *colors* of quarks (and an additional 6 antiquarks) must exist:
- a) **Up** ( $u$ ) quark has a rest energy of 360 MeV ( $1 \text{ MeV} = 10^6 \text{ eV}$ ) and a charge of  $+\frac{2}{3}e$ .
  - b) **Down** ( $d$ ) quark has a rest energy of 360 MeV and a charge of  $-\frac{1}{3}e$ .
  - c) **Charmed** ( $c$ ) quark has a rest energy of 1500 MeV and a charge of  $+\frac{2}{3}e$ .
  - d) **Strange** ( $s$ ) quark has a rest energy of 540 MeV and a charge of  $-\frac{1}{3}e$ .
  - e) **Top** ( $t$ ) quark has a rest energy of 170 GeV ( $1 \text{ GeV} = 10^9 \text{ eV}$ ) and a charge of  $+\frac{2}{3}e$ .
  - f) **Bottom** ( $b$ ) quark has a rest energy of 5 GeV and a charge of  $-\frac{1}{3}e$ .
5. Note that a proton is composed of 2  $u$  and a  $d$  quark and a neutron composed of an  $u$  and 2  $d$  quarks.
6. The theory on how quarks interact with each other is called **quantum chromodynamics**. One interesting result of this theory is that quarks cannot exist in isolation, they must always travel in groups of 2 to 3 quarks.



## The Standard Model of Particle Physics

Figure III-5: The Standard Model is the current best description of the subatomic world.

7. There are 2 additional terms that are used to describe particles — terms that describe the *spin* of a particle:
  - a) In quantum mechanics, a system of identical particles 1, 2, 3, ... is described by a **wave function**, which describes the spin of the particle.
  - b) A wave function must be either **symmetrical** (even) or **antisymmetrical** (odd) with respect to the interchange of coordinates of any pair of identical particles.
  - c) If symmetrical, the particles are called **bosons** and have zero or integer (*i.e.*, 0, 1, 2, 3, ...) spins.
  - d) If antisymmetrical, the particles are called **fermions** and have half-integer (*i.e.*,  $\frac{1}{2}$ ,  $\frac{3}{2}$ ,  $\frac{5}{2}$ , ...) spins.
  - e) An antisymmetrical wave function must vanish as 2 identical particles approach each other. As a result, 2 fermions

in the same quantum state exhibit a strong mutual repulsion  $\implies$  **Pauli Exclusion Principle**.

- f) No such restrictions exist for bosons.
- g) Leptons and baryons are fermions.
- h) Mesons and field particles (*i.e.*, photons) are bosons.

## E. History of the Universe

### 1. Singularity: the Big Bang Itself!

$$t = 0, \quad D = 0, \quad \rho = \rho_{\text{rad}} \rightarrow \infty, \quad T = T_{\text{rad}} \rightarrow \infty .$$

- a) If the Universe is closed, then a finite amount of mass-energy is located in a zero volume (like a black hole singularity).
- b) If the Universe is open or flat, then the Universe has an infinite total amount of mass-energy located in an infinite volume at this stage.
- c) We currently have no physics that can describe the history and events occurring in the Universe at this point. Perhaps if quantum mechanics and general relativity are ever combined (*i.e.*, **quantum gravity**) (see Figure III-6), we will have a physical theory that can describe the Universe here and explain why the Big Bang ever occurred.
- d) In the header lists for each of these eras,  $t$  represents time since the Big Bang,  $D$  the diameter of the Universe at time  $t$ ,  $\rho$  is the mass-energy density,  $T$  is the temperature, and later,  $z$  corresponds to the redshift. Each of these values are listed at the beginning and ending of each era.

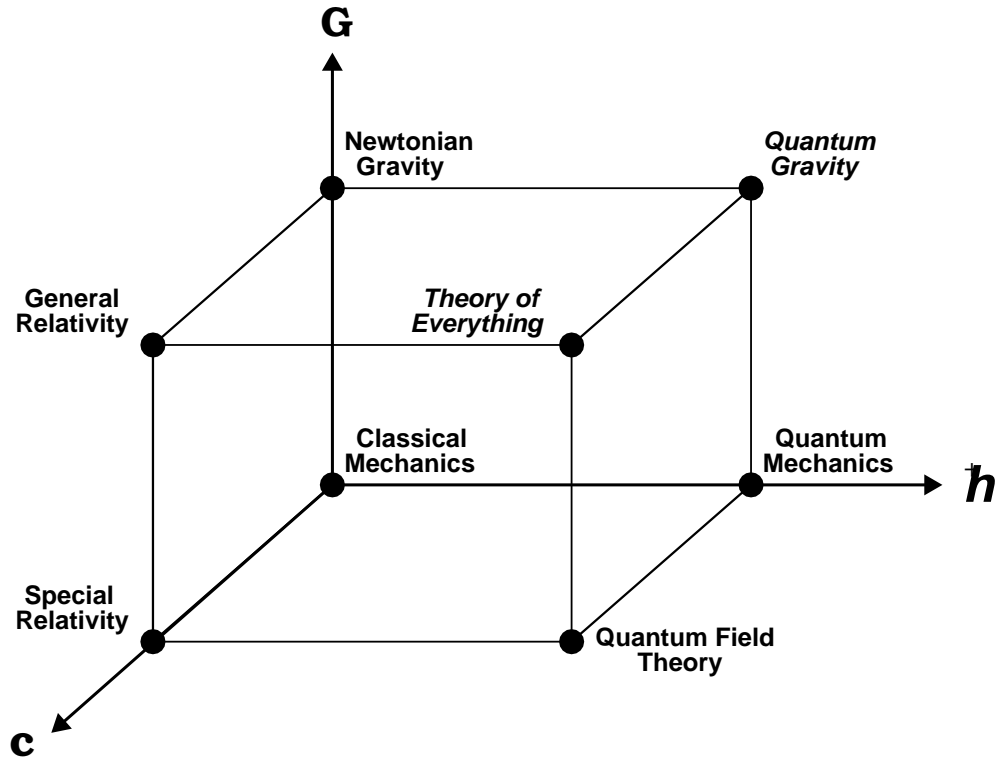


Figure III-6: The relationship of the various theories of the natural force with respect to the fundamental physical constants of the Universe. Note that those “theories” (actually hypotheses) in italics have not yet been confirmed.

## 2. Quantum Era

$$\begin{aligned}
 0 < t < 10^{-43} \text{ sec} &= t_P = \text{Planck Time}, \\
 0 < D < 10^{-33} \text{ cm} &= \ell_P = \text{Planck Length}, \\
 \rho = \rho_{\text{rad}} > 10^{90} \text{ gm/cm}^3, & \quad T = T_{\text{rad}} > 10^{32} \text{ K} .
 \end{aligned}$$

- a) The earliest time that can be addressed by current physical theory is the **Planck time**:

$$t_P \equiv \sqrt{\frac{\hbar G}{c^5}} = 5.39 \times 10^{-44} \text{ s} , \quad (\text{III-16})$$

where  $\hbar$  is the angular Planck’s constant ( $\hbar = h/2\pi$ ).

- b) In a Planck time, the speed of light crosses a distance called the **Planck length**:

$$\ell_P = t_P c \equiv \sqrt{\frac{\hbar G}{c^3}} = 1.62 \times 10^{-33} \text{ cm} . \quad (\text{III-17})$$

- c) The uncertainty principle from quantum mechanics tells us that the uncertainty in a particle's momentum times the uncertainty in position must be greater than  $\hbar$ . If we use the Schwarzschild radius of the early Universe as the position uncertainty, we can use conservation of energy to describe the **Planck mass**:

$$m_{\text{P}} \equiv \sqrt{\frac{\hbar c}{G}} = 2.18 \times 10^{-5} \text{ gm} . \quad (\text{III-18})$$

The Planck mass can be interpreted as the minimum mass that any primordial black holes can have if created in the Big Bang.

- d) Physics will not be understood in this Quantum Era until quantum effects are successfully included in gravity as described previously.
- e) During this time, it is speculated that all forces, including gravity, act as one force  $\implies$  the **Theory of Everything** (see Figure III-7).

### 3. GUT (Grand Unified Theory) Era

$$\begin{aligned} 10^{-43} \text{ sec} &< t < 10^{-34} \text{ sec} \\ 10^{-33} \text{ cm} &< D < 10^{-24} \text{ cm} \\ 10^{90} \text{ gm/cm}^3 &< \rho_{\text{rad}} < 10^{72} \text{ gm/cm}^3 \\ 10^{32} \text{ K} &< T < 10^{27} \text{ K} . \end{aligned}$$

- a) At the beginning of this era, gravity breaks from the unified force, following the equations of general relativity, and gets progressively weaker (see Figure III-7). This **symmetry breaking** acts like a phase transition of the Universe.

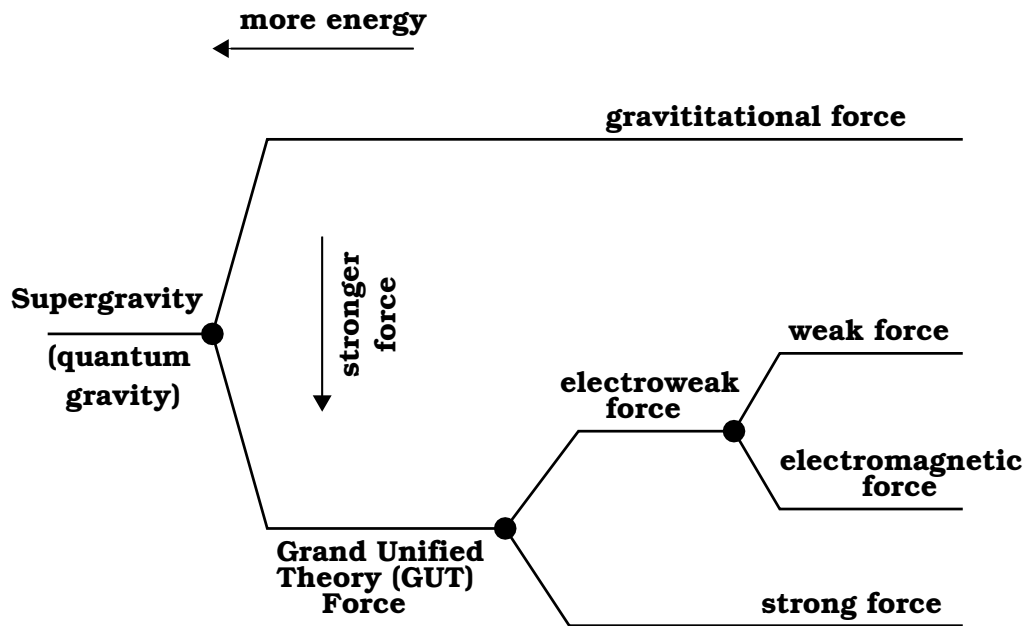


Figure III-7: The ‘Theory’ of Everything will describe the four natural forces as one force at very high energies (that is, early in the Universe). The four combined forces are called **unification** and are said to have symmetry. As it expands, the Universe cools (*i.e.*, loses energy) and at certain temperatures, forces start to decouple. Each *symmetry breaking* of the natural forces of the Universe, shown as ‘dots’ in this figure, correspond to the Universe changing its state. The decoupling of the GUT Force causes the Universe to expand exponentially instead of the standard linear expansion seen during the other epochs.

- b) During this time, the strong, weak, and electromagnetic forces act as one as described by the **Grand Unified Theory**.
- c) Temperature is so high that only field particles exist: gravitons, weakons (*i.e.*, intermediate vector bosons), photons, and gluons.
- d) At the end of this era, the strong force decouples from the **electroweak** force  $\implies$  the Universe goes through another phase transition.

## 4. Inflationary Era

$$\begin{aligned}10^{-34} \text{ sec} &< t < 10^{-32} \text{ sec} \\10^{-24} \text{ cm} &< D < 330 \text{ cm} \\10^{72} \text{ gm/cm}^3 &< \rho_{\text{rad}} < 10^{37} \text{ gm/cm}^3 \\10^{27} \text{ K} &< T < 10^{18} \text{ K} .\end{aligned}$$

- a) The electroweak-decoupling phase transition causes the Universe to expand exponentially from  $10^{-24}$  cm to 330 cm ( $10^{28}$  cm today).
- b) During this time, all baryonic matter is created from the primeval *soup* of field particles  $\implies$  individual **quarks** and **antiquarks** are made  $\implies$  matter (and antimatter) arise from the field energy particles via  $E = mc^2$ .
- i) Field particles all have integer spins  $\implies$  they are bosons.
- ii) The GUT predicts that baryon number conservation and charge & parity (CP) conservation can be violated occasionally.
- iii) These CP violations can cause slight asymmetries in decay rates of a given boson decaying to more stable particles:
- The *kaon*  $K$  can decay to a *pion*  $\pi$  via either
$$\begin{aligned}K &\rightarrow \pi^- + e^+ + \nu_e \\K &\rightarrow \pi^+ + e^- + \bar{\nu}_e .\end{aligned}$$
  - The first of these two reactions occurs slightly (but measurably) more frequently than the second.

- As such, it is possible to get slight asymmetries between matter and antimatter over time as these particles are made out of the field particle soup.
- c) During this time, the temperatures are too high for the strong force to connect the quarks together to make baryons and anti-baryons.
  - d) Due to CP violation, for every 30 million antiquarks, there are 30 million + 1 quarks by the end of this era.
  - e) The Universe resumes a linear expansion at the end of this era.

## 5. Quark Era

$$\begin{aligned}
 10^{-32} \text{ sec} &< t < 10^{-6} \text{ sec} \\
 330 \text{ cm} &< D < 10^9 \text{ cm} = 4D_{\oplus} \\
 10^{37} \text{ gm/cm}^3 &< \rho_{\text{rad}} < 10^{17} \text{ gm/cm}^3 \\
 10^{18} \text{ K} &< T < 10^{13} \text{ K} .
 \end{aligned}$$

- a) Forces between quarks act strangely: The farther away they get from each other, the stronger the force exerted (opposite of the direction of gravity). At the beginning of this era, the temperature is so high that quark motions can overcome this force and hence are not bound with each other.
- b) At  $t = 10^{-12}$  sec,  $T = 10^{15}$  K,  $\rho = 10^{24}$  gm/cm<sup>3</sup>, the electromagnetic and weak forces decouple  $\implies$  when this occurs, leptons start to form.
- c) This era ends when the temperature is cool enough for quarks to form bound states ( $T = 10^{13}$  K,  $\rho = 10^{17}$  gm/cm<sup>3</sup>) and become hadrons.

## 6. Hadronic Era

$$\begin{aligned}10^{-6} \text{ sec} &< t < 1 \text{ sec} \\10^9 \text{ cm} &< D < 10^{14} \text{ cm} = 15 \text{ AU} \\10^{17} \text{ gm/cm}^3 &< \rho_{\text{rad}} < 10^5 \text{ gm/cm}^3 \\10^{13} \text{ K} &< T < 10^{10} \text{ K} .\end{aligned}$$

- a) During this time, proton-antiproton pairs constantly annihilate and reform  $\implies$  hadrons are said to be in thermal equilibrium with photons (*i.e.*, the radiation field).
- b) At the end of this era,  $T = 10^{10}$  K,  $\rho = 10^5$  gm/m<sup>3</sup>, the energy density is too low to produce proton-antiproton pairs. These particles annihilate one last time — except there is a *slight* asymmetry between protons and antiprotons, for every  $10^9$  antiprotons there are  $10^9 + 1$  protons. The remaining protons have nothing to annihilate with and remain.

## 7. Lepton Era

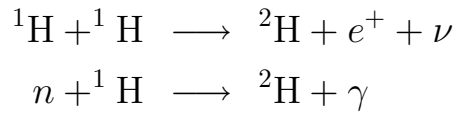
$$\begin{aligned}1 \text{ sec} &< t < 1 \text{ min} \\10^{14} \text{ cm} &< D < 10^{15} \text{ cm} = 150 \text{ AU} \\10^5 \text{ gm/cm}^3 &< \rho_{\text{rad}} < 10^4 \text{ gm/cm}^3 \\10^{10} \text{ K} &< T < 6 \times 10^9 \text{ K} .\end{aligned}$$

- a) Electron-positron pairs are still in equilibrium.
- b) At the end of this era,  $T = 6 \times 10^9$  K,  $\rho = 10^4$  gm/m<sup>3</sup>, the energy density becomes too low to make new electron-positron pairs  $\implies$  excess of electrons left over.

## 8. Nucleosynthesis Era

$$\begin{aligned}1 \text{ min} &< t < 3 \text{ min} \\10^{15} \text{ cm} &< D < 10^{19} \text{ cm} = 10 \text{ ly} \\10^4 \text{ gm/cm}^3 &< \rho_{\text{rad}} < 10^{-8} \text{ gm/cm}^3 \\6 \times 10^9 \text{ K} &< T < 10^7 \text{ K} .\end{aligned}$$

- a) Temperatures and densities exist such that H fuses into  ${}^2\text{H}$  (deuterium).



- b) Temperatures are too high to fuse  ${}^2\text{H} \implies$  it photodissociates as fast as it forms. He (helium) cannot be created — even though the temperature is high enough  $\implies$  this is called the **deuterium bottleneck**.
- c) When  $T < 10^9 \text{ K}$  ( $t \approx 100 \text{ sec}$ , 1.5 min),  ${}^2\text{H}$  no longer dissociates and fuses immediately into He  $\implies$  however, the Universe is now too cool to fuse  $\text{He} \rightarrow \text{C}$  (carbon).
- d) At  $t = 3 \text{ min}$ ,  $T < 10^7 \text{ K}$ ,  $\rho < 10^{-8} \text{ gm/cm}^3$ , and He production ceases. Since the last stages of the p-p chain are not as efficient as the first stage, an excess number of  ${}^2\text{H}$  (deuterium) is left over (and some  ${}^7\text{Li}$  [lithium-7] from the branch reactions of the full p-p chain)  $\implies$  the amount left over depends critically on the density of the Universe at that time. Since deuterium and lithium-7 are easily destroyed in the interior of stars, all of the  ${}^2\text{H}$  and  ${}^7\text{Li}$  we currently see in the Universe arose during this Nucleosynthesis Era.
- e) The future of the Universe is now set  $\implies$  all within the first 3 minutes!

## 9. Radiation Era

$$\begin{aligned} 3 \text{ min} &< t < 1000 \text{ yr} \\ 10^9 &< z < 2200 \\ 10^{19} \text{ cm} &< D < 10^{21} \text{ cm} = 1000 \text{ ly} = 300 \text{ pc} \\ 10^{-8} \text{ gm/cm}^3 &< \rho < 10^{-15} \text{ gm/cm}^3 \\ 10^7 \text{ K} &< T < 10^5 \text{ K} . \end{aligned}$$

- a) In this era,  $\rho_{\text{rad}} > \rho_{\text{matter}}$ .
- b) The temperature is still greater than  $10^5 \text{ K}$  which keeps hydrogen ionized.
- c) Since H is ionized, there are an abundance of free electrons which effectively *blocks* the flow of radiation (*i.e.*, photons)  $\implies$  Compton scattering and Thompson scattering.
- d) The Universe is completely *opaque* during this time.
- e) At the end of this era,  $\rho_m = \rho_{\text{baryon}} = \rho_B = \rho_{\text{rad}}$  (here we are assuming that all of the Universe's mass is in the form of baryons). With this information, we can now determine the total number of baryon particles in the Universe and the total mass of the baryons at this point. Both the total number and total mass remains constant from this point forward as the Universe expands.
  - i) The total mass of all the baryons present at the end of this era is equal to

$$M_B = V \rho_B = \frac{4\pi}{3} D^3 \rho_b . \quad (\text{III-19})$$

- ii) The baryon mass density can now be obtained with

$$\rho_B = \rho_{\text{rad}} = \frac{aT^4}{c^2} . \quad (\text{III-20})$$

Using the values for  $T$  listed above ( $a = 7.566 \times 10^{-15} \text{ erg/cm}^3/\text{K}^4$ ), we get  $\rho_b = 8.4 \times 10^{-16} \text{ gm/cm}^3$ .

- iii) Using the diameter of the Universe at the end of this era listed above, we get the total mass of the baryons as  $M_B = 3.5 \times 10^{48} \text{ grams} = 2 \times 10^{15} M_\odot$ .

- iv) At this point in the Universe, 90% of all the particles are hydrogen nuclei (protons) and 10% are helium nuclei (2 protons + 2 neutrons). With this information, we can determine the total number of baryons that were created from the Big Bang:

$$\begin{aligned} n_B &= 0.90 \frac{M_B}{m_{\text{H}}} + 0.10 \frac{M_B}{m_{\text{He}}} = \left( \frac{0.9}{m_{\text{H}}} + \frac{0.1}{m_{\text{He}}} \right) M_B \\ &= 1.9 \times 10^{72} \text{ baryons.} \end{aligned}$$

- f) We know that the COBE and WMAP microwave maps of the sky show temperature (hence density) variations (*i.e.*, fluctuations) on the cosmic microwave background. We now ask the question that if these density fluctuations existed during the radiation era, will they become unstable and collapse under their own weight?

- i) To answer this, we just need to calculate the Jeans' mass of the Universe during this era.
- ii) Kolb and Turner, in their *The Early Universe* textbook, give the Jeans' mass of baryons for an expanding universe during the radiation era as

$$M_{B-J}^r = 5.4 \times 10^{18} \Omega_B h^2 T_{\text{eV}}^{-3} M_\odot , \quad (\text{III-21})$$

where  $T_{\text{ev}}$  is the temperature of the radiation field expressed in units of eV ( $1 \text{ eV} = 1.1605 \times 10^4 \text{ K}$ ).

- iii) If we use the temperature calculated for the cross-over time and use a temperature slightly higher than this (say  $1.50 \times 10^5 \text{ K} = 12.9 \text{ eV}$ ) for the temperature of the radiation field just prior to the radiation-dominated to matter-dominated cross-over, setting  $\Omega_B = \Omega_\circ$  gives a Jeans' mass of

$$M_{B-J}^r = \frac{5.4 \times 10^{18} M_\odot}{(12.9)^3} = 2.5 \times 10^{15} M_\odot . \quad (\text{III-22})$$

- iv) As can be seen,  $M_B < M_{B-J}^r$ , so no gravitational instabilities will grow and any density fluctuations that exist will not result in a collapse.

## 10. Matter Era

$$\begin{aligned} 1000 \text{ yr} &< t < \text{present} = 13.7 \text{ Gyr} \\ 2200 &< z < 0 \\ 10^{21} \text{ cm} &< D < 10^{28} \text{ cm} = 13.7 \text{ Gly} = 4.2 \text{ Gpc} \\ 10^{-15} \text{ gm/cm}^3 &< \rho < 3 \times 10^{-30} \text{ gm/cm}^3 \\ 10^5 \text{ K} &< T < 2.7 \text{ K} . \end{aligned}$$

- a) Here we are assuming that *present* is at  $t = 13.7 \times 10^9$  (13.7 billion) years.
- b) Matter begins to dominate radiation in this era:  $\rho_{\text{matter}} > \rho_{\text{rad}}$ .
- c) H (hydrogen) becomes completely neutral when  $T < 3000 \text{ K}$  ( $t = 380,000$  years,  $z = 1100$ )  $\implies$  the Universe becomes transparent to light since the opacity from electron scattering drops to zero!

- i) We see this epoch today as the 2.7 K background radiation  $\implies$  visible light when emitted at  $z = 1100$ , redshifted today ( $z = 0$ ) to microwave radiation.
  - ii) This time is called the **recombination time** of the Universe.
  - iii) This is what we are seeing when we observe the 2.7 K background.
- d) Inhomogeneities in the matter and radiation fields begin to grow due to gravitational instabilities just after recombination.
- i) Kolb and Turner, in their *The Early Universe* textbook, also give the Jeans' mass of baryons for an expanding universe during the time just after recombination as

$$M_{B-J}^m = 1.3 \times 10^5 (\Omega_B h^2)^{-1/2} \left( \frac{z}{1100} \right) M_\odot . \quad \text{(III-23)}$$

- ii) Since  $z < 1100$  past the recombination time,  $M_{B-J}^m < 1.3 \times 10^5 M_\odot$  for the remainder of the matter era. As such,  $M_B \gg M_{B-J}^m \rightarrow$  the mass of baryons will become gravitationally unstable which will cause density fluctuations to grow and collapse to occur. Note that due to this large inequality, the collapse occurs very rapidly.
- e) Galaxies begin to “light-up” as the density inhomogeneities collapse to a high enough value that star formation begins.

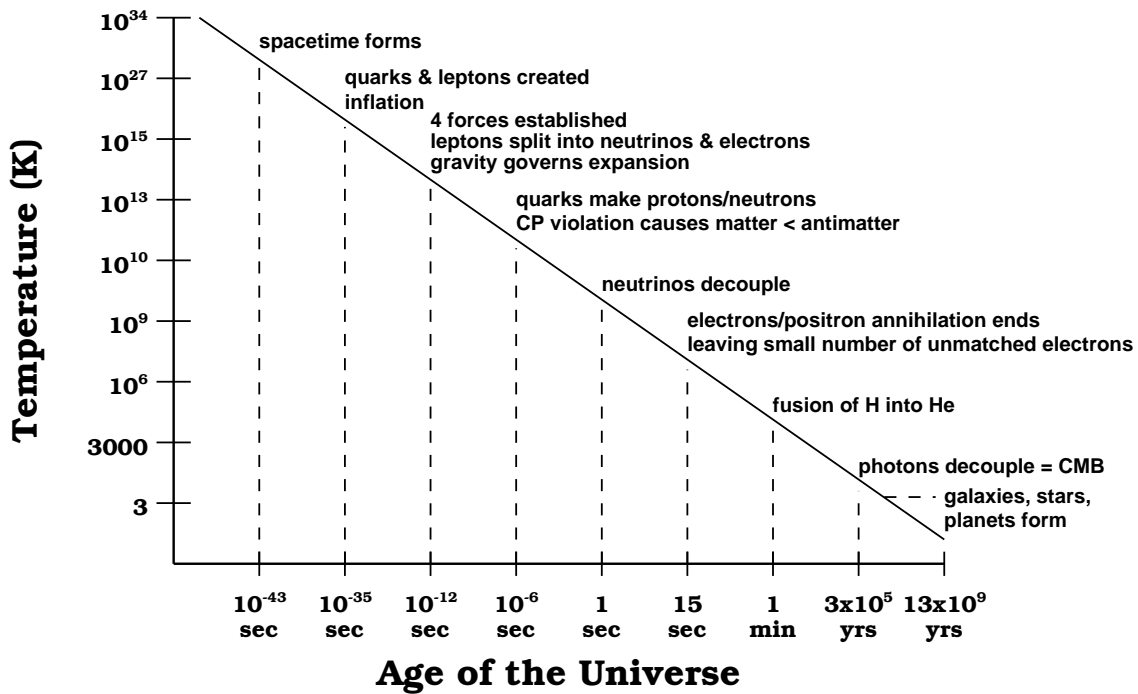


Figure III-8: Graphical representation of the history of the Universe.

- i) It is at this time that the Population III stars (*i.e.*, no metallicity) begin to form in earnest from the IGM and ISM.
- ii) This occurs at approximately  $z = 20$ ,  $t = 1 \times 10^8$  years after the Big Bang.
- f) Galaxies begin to cluster at  $z \approx 10$ ,  $t = 3 \times 10^8$  years.
- g) The first Population II (low metallicity) stars form in our Galaxy out of material expelled from the Population III stars at  $z \approx 4.5$ ,  $t = 8 \times 10^8$  years after the Big Bang.
- h) Quasars (described in the next section) become active and Population II stellar formation rates begin to drop in the Milky Way at  $z \approx 3$ ,  $t = 1 \times 10^9$  years.

- i) Population I stars begin to form in Milky Way at  $z \approx 1$ ,  $t = 4 \times 10^9$  years.
- j) The Sun and solar system form when  $z = 0.02$  and  $t = 9 \times 10^9$  years after the Big Bang.
- k) Life begins on Earth around  $t = 10 \times 10^9$  years after the Big Bang, and the first primates arise at  $t = 13.696 \times 10^9$  years after the Big Bang.
- l) Presently the Universe is at of age of  $13.7 \times 10^9$  (13.7 billion) years (*i.e.*,  $z = 0$ ).