

Physics 2018: Great Ideas in Science: The Fall 2008 Physics Module

Dr. Donald G. Luttermoser
East Tennessee State University

Edition 2.0

Abstract

These class notes are designed for use of the instructor and students of the course **Physics 2018: Great Ideas in Science**. This edition was last modified for the Fall 2008 semester.

I. Thermodynamics: Equilibrium vs. Non-Equilibrium

A. The Nature of Physics: Behavior and composition of matter and energy and their *interactions*.

1. 2 main branches:

a) **Classical Physics:**

i) Classical Mechanics (the study of motion and gravity).

ii) Thermodynamics (the study of heat motion).

iii) Fluid Mechanics (the study of fluids).

iv) Electromagnetism (the study of electricity and magnetism).

v) Optics (the study of light/glass interactions).

vi) Wave Mechanics (the study of oscillatory motion).

b) **Modern Physics:**

i) Special Relativity and General Relativity (modern motion and gravity).

ii) Quantum Mechanics (also called Atomic Physics).

iii) Nuclear Physics.

iv) Statistical Mechanics (thermodynamics in terms of probabilities).

v) Condensed Matter (once called Solid State Physics).

2. Matter *moves* (*i.e.*, follows trajectories) as a result of a force being applied to it.

a) **Contact forces:** Force exerted through a collision as described by Newton's 2nd law of motion: $F = ma$.

b) **Field (or natural) forces:** Force exerted on an object from its location in some natural potential field. There are 4 field forces in nature:

Interaction	Relative Strength	Range
Strong [†]	1	10^{-15} m
Electromagnetic ^{†‡}	10^{-2}	∞
Weak ^{†‡}	10^{-6}	10^{-17} m
Gravitational	10^{-43}	∞

† - Under high energies, the electromagnetic and weak forces act as one — the Electroweak force.

‡ - Under even higher energies, all of the natural forces (except gravity) also may act as one, as described by the Grand Unified Theory.

B. Useful Definitions of Motion.

1. The **displacement** of an object is defined as the change in its position.

a) It is given by the difference between its final and initial coordinates:

$$\boxed{\Delta x \equiv x_f - x_i} = \text{displacement.} \quad (\text{I-1})$$

b) Displacement is a vector quantity \implies has direction and magnitude:

$$\Delta \vec{x} \equiv \vec{x}_f - \vec{x}_i . \quad (\text{I-2})$$

2. Velocity.

- a) The **average velocity**, \bar{v} , is defined as the displacement divided by the time interval during which the displacement occurred:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

or in vector notation:

$$\boxed{\vec{v} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}_f - \vec{x}_i}{t_f - t_i} .} \quad (\text{I-3})$$

- b) The average velocity of an object during the time interval t_i to t_f is equal to the slope of the straight line joining the initial and final points on a graph of the position of the object plotted versus time.
- c) When plotting the value of a variable as a function of some other variable, the path on that plot that the object takes is called a **trajectory**.
- d) The **instantaneous velocity**, \vec{v} , is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small.

$$\boxed{\vec{v} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{x}}{\Delta t}} \equiv \frac{d\vec{x}}{dt} . \quad (\text{I-4})$$

3. Acceleration.

- a) The **average acceleration**, \bar{a} , during a given time interval is defined as the change in velocity divided by that time interval during which the change occurs:

$$\boxed{\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} .} \quad (\text{I-5})$$

- b) The **instantaneous acceleration** of an object at a certain time equals the slope of the velocity-time graph at

that instant in time (*i.e.*, tangent line of $v(t)$ at that point).

$$\boxed{\vec{a} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t}} \equiv \frac{d\vec{v}}{dt} = \frac{d}{dt} \left(\frac{d\vec{x}}{dt} \right) = \frac{d^2\vec{x}}{dt^2} . \quad (\text{I-6})$$

C. Newton's Laws of Motion.

1. Newton's 1st Law: Law of Inertia: An object at rest remains at rest, and an object in motion continues in motion with a constant velocity, unless it is acted upon by an external force.

- a) **Inertia** is the resistance that matter has to changes in motion.
- b) The **mass** of an object measures that object's inertia.
 \implies Mass is nothing more than a measure of matter's resistance to changes in motion.

2. Newton's 2nd Law: The acceleration (a) of an object is directly proportional to the resultant force (F) acting on it and inversely proportional to its mass (m). The direction of the acceleration is the same direction as the resulting force.

$$\boxed{\sum \vec{F} = m \vec{a} .} \quad (\text{I-7})$$

- a) **This is arguably the most important equation in physics and possibly all of science.**
- b) Force is measured in **newtons** in the SI system:

$$\boxed{1 \text{ N} \equiv 1 \text{ kg}\cdot\text{m}/\text{s}^2,} \quad (\text{I-8})$$

\implies or in the cgs system:

$$1 \text{ dyne} \equiv 1 \text{ g}\cdot\text{cm}/\text{s}^2 = 10^{-5} \text{ N},$$

\implies or in the English system:

$$1 \text{ lb} \equiv 1 \text{ slug} \cdot \text{ft}/\text{s}^2 = 4.448 \text{ N}.$$

- 3. Newton's 3rd Law:** If 2 bodies interact, the magnitude of the force exerted on body 1 by body 2 is equal to the magnitude of the force exerted on body 2 by body 1, and these forces are in opposite direction to each other.
- a) Another way of saying this is “for every action, there is an opposite reaction.”
 - b) Newton's 3rd law is nothing more than the *conservation of linear momentum*.

D. Equilibrium in Motion.

1. Keywords that tell you an object is in (motion) **equilibrium** \implies no acceleration.
 - a) Body is at rest (not changing position).
 - b) Body is static (not changing in time).
 - c) Body is in steady state (no acceleration).
2. When objects are in (motion) equilibrium:

$$\boxed{\sum \vec{F} = 0} \quad (\text{I-9})$$

or in component format (\sum means summation of all forces):

$$\begin{aligned}\sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 .\end{aligned}$$

E. Kinematics vs. Dynamics.

1. The word **mechanics** in physics means the *study of motion*.
 - a) The study of motion without regards to its causes is called **kinematics**.
 - b) The study of the causes of motion is called **dynamics**.
2. Objects that move in **uniform motion** (*i.e.*, at a **constant velocity**) have the following characteristics:
 - a) Since $\Delta v = 0$ for uniform motion, objects with such motion are **not accelerating** (remember $a = \Delta v / \Delta t$).
 - b) From Newton's 2nd law (see Eq. I-7), zero acceleration means that the total force in all directions (*i.e.*, the vector sum of all forces acting on the object) equals zero.
 - c) As such, if an object is moving in uniform motion and there is an applied force causing this motion, there must be a *retarding* force (such as friction) acting in the opposite direction.
 - d) Objects (and fluids) that are moving in uniform motion are said to be in **static equilibrium**. We will have more on equilibrium later.

F. State Variables: Quantities that describe the condition or **state** of a system.

1. **Pressure:** $P = F/A$ [SI units: Pa = N m⁻²] or [cgs units: dyne cm⁻²].
2. **Temperature:** A measure of the amount of heat in a system [K] or [°C]. More precisely, it is a measure of the average velocity of the particles in matter.

3. Volume (or Density):

- a) **Mass Density:** ρ ('rho') \equiv amount of mass per unit volume [SI units: kg m^{-3}] or [cgs units: g cm^{-3}].
- b) **Number Density:** N ('capital N') \equiv number of particles per unit volume [SI units: m^{-3}] or [cgs units: cm^{-3}]. Note that one could include 'atoms,' 'molecules,' or 'particles' in the numerator of the units for this parameter, but these terms are not really *units* (like 'radian' and angular 'degree'), and as such, I have not included any of these labels in the 'units' for number density, though their inclusion are implied.
- c) The total number of particles and volume can be expressed separately as well:
 - i) **Volume:** $V \equiv$ Total 'space' that the gas fills [SI units: m^3] or [cgs units: cm^3]
 - ii) **Particle Number:** n ('small n') \equiv number of particles present \implies typically measured in **moles** \rightarrow the amount of material whose mass in grams is numerically equal to the molecular mass of the substance (*i.e.*, the atomic mass of nitrogen N is 14 since its nucleus is composed of 7 protons and 7 neutrons, the molecular mass of nitrogen N_2 is (14+14=) 28 \rightarrow a mole of atomic nitrogen is 14 grams and a mole of molecular nitrogen is 28 grams) [mol].

G. Thermodynamics.

1. In this class, we are interested in *changes* in nature. The concept of change is at the heart of the area of physics known as thermodynamics which is concerned with the relationships between

heat and work. **Thermodynamics** is essentially the study of the motion of heat.

2. The First Law of Thermodynamics.

- a) In words: **The change in internal energy of a system equals the difference between the heat taken in by the system and the work done on the system.**
- b) When an amount of heat Q is added to a system, some of this added energy remains in the system increasing its internal energy by an amount ΔU .
- c) The rest of the added energy leaves the system as the system does work W .
- d) Mathematically, note that

$$\Delta U = U_f - U_i ,$$

and the First Law states

$$\boxed{\Delta U = Q + W .} \quad (\text{I-10})$$

- e) In thermo, there will always be two specific regions in which we will be interested in:
 - i) The **system** — the region of interest where we wish to know the state parameters (*e.g.*, P = pressure, T = temperature, and V = volume). Thermodynamic variables relating to the system will remain ‘unsubscripted’ in these notes (*i.e.*, W = work on the system).
 - ii) The **environment** (also called the *universe*) — the region that contains the system. Note that

some scientists (including me) prefer to use the word *environment* over *universe* since in astronomy the word **Universe** means all of the cosmos and not just the immediate (*i.e.*, nearby) surroundings which is what thermo means by “universe.” Thermodynamic variables relating to the environment will be labeled with the ‘env’ subscript in these notes (*i.e.*, W_{env} = work on the environment).

iii) Note that work done on the system W is the exact opposite of work done on the environment:

$$W = -W_{\text{env}} .$$

f) Note that we will often be interested in systems that are completely isolated from the environment. Such a system is called a **closed system**.

3. The Second Law of Thermodynamics — The Classical Description.

a) The second law of thermodynamics deals with how heat flows. It is essentially a description of *change*.

i) **Change:** To make different the form, nature, and content of something.

ii) Change has, over the course of time and throughout all space, brought forth, successively and successfully, galaxies, stars, planets, and life.

iii) Evidence for change is literally everywhere.

iv) Much of the change is subtle, such as when the Sun fuses hydrogen into helium sedately over bil-

lions of years or when the Earth's tectonic plates drift sluggishly across the face of our planet over those same billions of years.

- v) Indeed, our perception of time is nothing more than our noticing changes on Earth and in the Universe as a whole.

- b) There are two classical formulations of this law (both essentially mean the same thing):
 - i) **Clausius statement of the second law:** Heat cannot, by itself, pass from a colder to a warmer body.

 - ii) **Kelvin-Planck statement of the second law:** It is impossible for any system to undergo a cyclic process whose *sole* result is the absorption of heat from a single reservoir at a single temperature and the performance of an equivalent amount of work.

- c) The 2nd law specifies the way in which *available energy* (also called “usable energy,” “free energy,” or “potential energy”) change occurs.
 - i) This law's essence stipulates that a price is paid each time energy changes from one form to another.

 - ii) The price paid (to Nature) is a loss in the amount of available energy capable of performing work of some kind in the future.

 - iii) We define here a new term to describe this decrease of available energy: *entropy*, S . It is derived

from the Greek word *tropae* which means “transformation.” We will have more to say about entropy next lecture.

- d) The second law of thermodynamics is different from the laws of mechanics. It does not describe the interactions between individual particles, but instead describes the overall behavior of collections of many particles.
 - i) In classical mechanics, an event is symmetric in time \implies the laws are satisfied whether we run the experiment forward or backward in time.
 - ii) The second law tells us about the sequence, or order, in which events naturally take place \implies **the second law of thermodynamics shows us the direction in which time progresses.**

4. The Second Law of Thermodynamics — The Probabilistic Description.

- a) In the late 1800s, Boltzmann showed that an increase in entropy of a system corresponds to an increased degree of *disorder* in the atoms or molecules composing the substance.
 - i) This realization lead to the creation of a new area of physics known as **statistical mechanics** which rewrites the laws of thermodynamics in a probabilistic formalism.
 - ii) Boltzmann rewrote the entropy equation definition in terms of a probability equation:

$$S = k_B \ln W , \quad (\text{I-11})$$

where k_B is Boltzmann's constant, 'ln' is the natural logarithm of base 'e', and W is the number of different arrangements of microscopic states (*i.e.*, positions, velocities, compositions, and any various arrangements of quantum properties). **Note that W here does not mean work!**

iii) Boltzmann actually has Eq. (I-11) carved on his gravestone in Vienna!

iv) W is a measure of the inverse probability, p , of the occurrence of the possible microscopic states a system can have.

v) That is, $p = 1/W = 1$ for a 100% chance \implies Then it is completely certain that a process will occur when $p = 1$.

vi) As a result, we could also write the definition of entropy (*i.e.*, Eq. I-11) as

$$S = k_B \ln \left(\frac{1}{p} \right) = -k_B \ln p . \quad (\text{I-12})$$

b) With the help of Eq. (I-12) we can see the second law implies that any isolated system naturally tends towards an equilibrium state of minimum microscopic probability — namely, a uniformity of temperature, pressure, chemical composition, and so on.

i) Since ordered molecular states (for example, where molecules in one part of the system have one property value, but those in the remaining part have another) are less probable than those of random or disordered states.

- ii) Boltzmann's law of entropy then signifies that ordered states tend to degenerate into disordered ones in a closed system.
- c) As can be seen, the concept of "disorder" is very difficult since it requires a detailed knowledge of probability and statistics.
- i) The best way to learn about probabilities is through example. Let's say we have one die from a set of dice. There are 6 sides with dots imprinted on the sides relating to the numbers 1, 2, 3, 4, 5, and 6.

- ii) The probability of one number (say '4') coming up is

$$p = \frac{n}{N} = \frac{\text{a given state}}{\text{total number of states}} , \quad (\text{I-13})$$

and for this case, $n = 1$ and $N = 6$, so

$$p = \frac{\text{one side}}{\text{total number of sides}} = \frac{1}{6} = 0.167 ,$$

or a 16.7% chance that we would role the die with a '4' landing on top.

- iii) The probability of an even number (2, 4, or 6) landing on top is ($n = 3$ and $N = 6$)

$$p = \frac{3}{6} = 0.50 ,$$

or a 50% chance to role such a number.

- d) Through this concept of entropy, we can rewrite the second law of thermodynamics as any of the following statements:

- i) The entropy of the Universe as a whole increases in all natural processes.

- ii) Isolated systems tend towards greater disorder and entropy is a measure of that disorder.
 - iii) In a closed system, entropy increases over time \implies less and less energy can be converted into work.
 - iv) All of these statements are probabilistic in nature \rightarrow on average this is true.
- e) Note that the second law written in this probabilistic way can be violated locally \implies entropy can decrease locally. Only over the whole isolated (or closed) system over a long enough period of time, will necessitate an increase in entropy.
- i) Note that on the Earth, entropy decreases all the time at the expense of an increase of entropy of the Sun.
 - ii) As such, the second law of thermodynamics cannot be used as proof against the theory of biological evolution as some people have suggested.
- f) The bottom line is that we no longer regard things as “fixed” or “being,” or even that they “exist.” Instead, everything in the Universe is “flowing,” always in the act of “becoming.” All entities — living and non-living alike — are permanently changing.

H. Transport of Heat Energy.

1. Before discussing equilibrium vs. non-equilibrium states, we need to see how heat moves through matter and/or space.
2. Thermal energy (*i.e.*, heat) can only flow by one of three different mechanisms: **conduction**, **convection**, and **radiation transport**.

3. Heat Transfer by Conduction.

a) **Conduction** is the process by which heat is transferred via collisions of internal particles that make up the object \implies *individual* (mass) particle transport.

i) Heat causes the molecules and atoms to move faster in an object.

ii) The hotter molecules (those moving faster) collide with cooler molecules (those moving slower), which in turn, speeds up the cooler molecules making them warm.

iii) This continues on down the line until the object reaches equilibrium.

b) The amount of heat transferred ΔQ from one location to another over a time interval Δt is

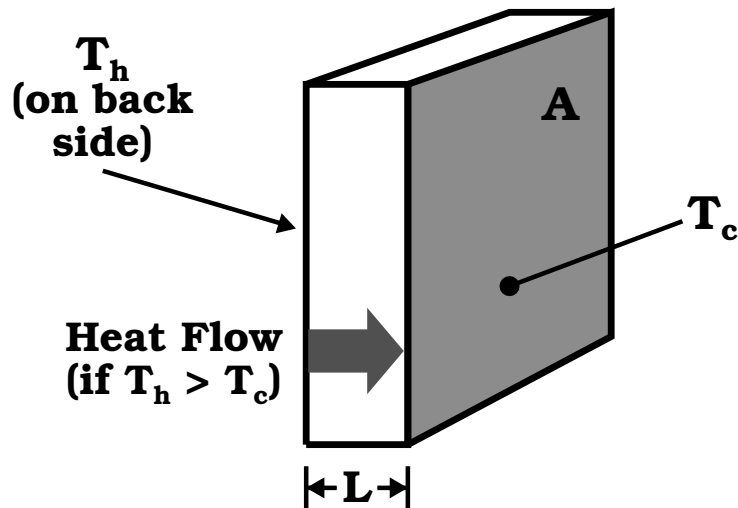
$$\boxed{\Delta Q = \mathcal{P} \Delta t} . \quad (\text{I-14})$$

i) $\mathcal{P} \equiv$ heat transfer rate.

ii) \mathcal{P} is measured in **watts** when Q is measured in Joules and Δt in seconds.

- iii) As such, \mathcal{P} is the same thing as power since they are both measured in the same units.
- c) Heat will only flow if a temperature difference exists between 2 points in an object.
- i) For a slab of material of thickness L and surface area A , the heat transfer rate for conduction is

$$\mathcal{P}_{\text{cond}} = \frac{\Delta Q}{\Delta t} = KA \left(\frac{T_h - T_c}{L} \right). \quad (\text{I-15})$$



- ii) T_h is the temperature of the hotter side and T_c is the temperature of the cooler side.
- iii) $K \equiv$ **thermal conductivity** of the material $\implies [K] = \text{J}/(\text{s m } ^\circ\text{C}) = \text{W}/\text{m}/^\circ\text{C}$.
- iv) The larger K is, the better the material is in conducting heat.
- v) The smaller K is, the better the material is as a thermal insulator.

- vi) The effectiveness of thermal insulation is rated by another quantity \implies **thermal resistance**, R , where

$$R \equiv \frac{L}{K}, \quad (\text{I-16})$$

where in the U.S., $[R] = \text{ft}^2 \text{ h } ^\circ\text{F}/\text{BTU}$ ('h' = hour), and elsewhere, $[R] = \text{m}^2 \text{ } ^\circ\text{C}/\text{W}$.

4. Heat Transfer by Convection.

- a) When an ensemble of hot particles move in bulk to cooler regions of a gas or liquid, the heat is said to flow via **convection**.
- b) Boiling water and cumulus clouds are 2 examples of convection.
- c) Convection is complicated requiring graduate-level physics and math to describe it.
- i) Convection will occur if an instability occurs in the gas or liquid.
- ii) The **mixing-length theory** is often used to describe convection: *The heat of a gas/liquid bubble will give up its heat energy to the cooler surroundings after the bubble has traveled one **mixing length**.*

5. Heat Transfer by Radiation.

- a) Of all the heat energy transport mechanisms, only radiation does not require a medium \implies it can travel through a vacuum.

- b) The rate at which an object emits radiant energy is given by the **Stefan-Boltzmann Law**:

$$\boxed{\mathcal{P}_{\text{em}} = \sigma A e T^4 .} \quad (\text{I-17})$$

- i) $\mathcal{P}_{\text{em}} \equiv$ power radiated (emitted) [watts].
- ii) $\sigma \equiv$ Stefan-Boltzmann's constant = 5.6696×10^{-8} W/m²/K⁴.
- iii) $A \equiv$ surface area of the object [m²].
- iv) $e \equiv$ emissivity [unitless] ($e = 1$ for a perfect absorber or emitter).
- v) $T \equiv$ temperature [K].
- c) A body also can absorb radiation. If a body absorbs a power of radiation \mathcal{P}_{abs} , it will change its temperature to T_{\circ} .
- i) The net power radiated by the system is then

$$\mathcal{P}_{\text{rad}} = \mathcal{P}_{\text{net}} = \mathcal{P}_{\text{em}} - \mathcal{P}_{\text{abs}} \quad (\text{I-18})$$

or

$$\mathcal{P}_{\text{rad}} = \sigma A e T^4 - \sigma A e T_{\circ}^4 ,$$

$$\boxed{\mathcal{P}_{\text{rad}} = \sigma A e (T^4 - T_{\circ}^4) .} \quad (\text{I-19})$$

- ii) In astronomy, the total power radiated by an object over its entire surface is called the **luminosity**, $L = \mathcal{P}_{\text{rad}}$, of the object. Since the amount of energy falling on the surface of the Sun (or any isolated star) from interstellar space is negligible to that of the power radiated, $L = \mathcal{P}_{\text{em}}$ for isolated stars.

- iii) If an object is in **equilibrium** with its surroundings, it radiates and absorbs energy at the same rate \implies its temperature remains constant \implies this **radiative equilibrium** results in the object being in **thermal equilibrium**:

$$\mathcal{P}_{\text{rad}} = 0 \quad \implies \quad T = T_{\circ} ,$$

where T_{\circ} is the temperature of the surroundings.

- d) An **ideal absorber** is defined as an object that absorbs all of the energy incident upon it.

- i) In this case, emissivity (e) = 1.

- ii) Such an object is called a **blackbody** (see next section):

$$\boxed{\mathcal{P}_{\text{bb}} = \sigma AT^4 .} \quad (\text{I-20})$$

- iii) Note that a blackbody radiator can be any color (depending on its temperature \implies red blackbodies are cooler than blue blackbodies), it does not appear “black” (unless it is very cold).

- iv) The **energy flux** of such a radiator is

$$F_{\text{bb}} = \frac{\mathcal{P}_{\text{bb}}}{A} = \sigma T^4 . \quad (\text{I-21})$$

- v) This radiative flux results from the condition that in order to be in thermal equilibrium, the heat gained by absorbing radiation must be (virtually immediately) radiated away by the object.

- vi) This is not the same thing as *reflecting* the radiation off of the surface (which does not happen in

a blackbody). The incident radiation does get “absorbed” by the atoms of the object and deposited in the thermal “pool.” It is just that this radiation immediately gets re-emitted just after absorption.

I. Blackbody Radiation.

1. Late in the 1800s and in the early part of the 20th century, Boltzmann, Planck, and others investigated E/M radiation that was given off by hot objects.
 - a) In general, matter can **absorb** some radiation (*i.e.*, photons converted to thermal energy), **reflect** some, and **transmit** some of the energy.
 - b) The color of cool objects, objects that don’t emit their own visible light, is dictated by the wavelengths of light they either reflect, absorb, or transmit.
 - i) A blue sweater is “blue” because the material reflects blue light (from either room lights or the Sun) more effectively than the other colors of the rainbow.
 - ii) Coal is black because it absorbs visible light and reflects very little.
 - iii) Glass is transparent because visible light is almost completely transmitted through the glass with little absorption and reflection.
2. To make the physics a little less complicated, these scientists invented the concept of an **ideal** or **perfect radiator**.
 - a) A hypothetical body that completely absorbs every kind of E/M radiation that falls on it.

- b) This absorption continues until an equilibrium temperature is reached.
 - c) At that point, all incoming radiation is immediately re-radiated away as soon as it is absorbed (note that this is not the same as being reflected).
 - d) Such a perfect radiator or absorber is called a **blackbody** (as previously mentioned).
3. As we have already seen, a blackbody radiator has an **energy flux** F (defined as the energy emitted per unit area each second and related to the intensity) that is radiated away which is proportional to the 4th power of temperature via Eq. (I-21).
- a) Note that the temperature in Eq. (I-21) is in units of Kelvin (K) and is the equilibrium or **effective temperature**.
 - b) Since the energy flux or brightness of a blackbody radiator is proportional to the fourth power of temperature, even a small increase in an object's temperature will cause a substantial increase in brightness.
4. The total brightness, or **luminosity** (L), of a blackbody is just the flux integrated over all of the surface of the object. For a spherical object, the surface area is $4\pi R^2$, where R is the radius of the spherical blackbody, so the luminosity is

$$L = 4\pi R^2 F = 4\pi \sigma R^2 T^4. \quad (\text{I-22})$$

Note that if we treat stars as blackbodies, we can eliminate the constants in the above equation by dividing both sides by *solar* values:

$$\frac{L}{L_{\odot}} = \frac{4\pi \sigma R^2 T^4}{4\pi \sigma R_{\odot}^2 T_{\odot}^4}$$

$$\frac{L}{L_{\odot}} = \left(\frac{R}{R_{\odot}}\right)^2 \left(\frac{T}{T_{\odot}}\right)^4. \quad (\text{I-23})$$

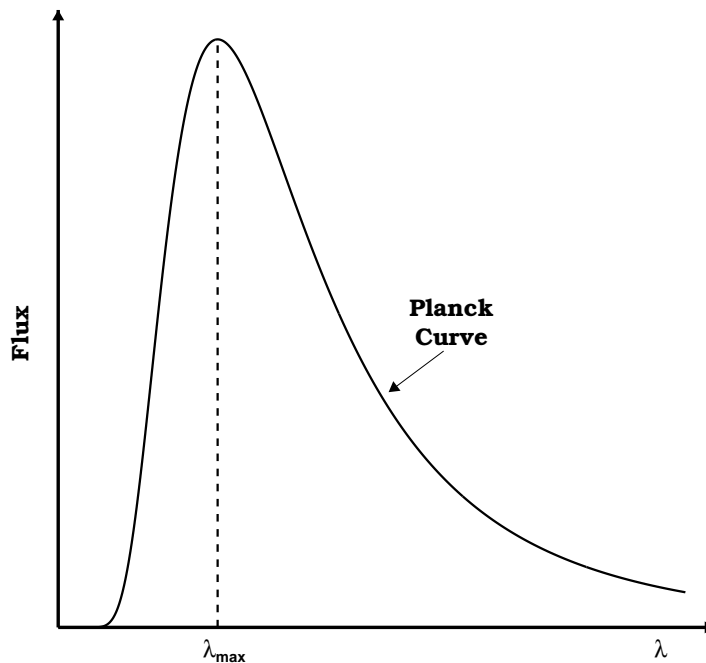
5. In 1893, Wien discovered a simple relationship between T of a blackbody and the wavelength where the maximum amount of light is emitted \implies **Wien's displacement law** (usually called Wien's law for short):

$$\lambda_{\max} = \frac{2.897 \times 10^{-3} \text{ m} \cdot \text{K}}{T}, \quad (\text{I-24})$$

or in the other wavelength units we have discussed:

$$\begin{aligned} \lambda_{\max} &= \frac{0.2897 \text{ cm} \cdot \text{K}}{T} \\ &= \frac{2.897 \times 10^6 \text{ nm} \cdot \text{K}}{T} \\ &= \frac{2.897 \times 10^7 \text{ \AA} \cdot \text{K}}{T}. \end{aligned}$$

6. Blackbody radiators, being in thermal equilibrium, emit continuous spectra that are called **Planck curves**:



7. From the Stefan-Boltzmann's and Wein's Displacement Laws, we see that as an object get cooler, it gets redder and fainter. And, as an object gets hotter, it gets bluer and brighter.

J. Equilibrium vs. Non-Equilibrium.

1. Definitions.

- a) The simplest definition of **equilibrium** is being in a **state of balance**.
- b) States in equilibrium have the lowest internal energy possible for that given state.
- c) As a result, **non-equilibrium** simply means that an **imbalance** must be present for a given state, or two opposing forces being unequal.

2. Types of Equilibria.

- a) **Equilibrium Forces.** These are forces that counteract other forces that cause motion either forcing an object to remain at rest or continuing its uniform motion (*i.e.*, no acceleration).
 - i) Say we have an object resting on a desk. The weight of the object, or the gravitational force downward, is balanced by the force of the desk pushing upward (as there must be to keep the object from moving in response to gravity). This upward force is called the **normal force**. "Normal" because this force is perpendicular to the surface of the desk. This normal force results from the electromagnetic forces between the atoms and molecules in the desk that make it rigid (*i.e.*, a solid).

ii) If an object hangs from wires or ropes, another equilibrium force (*i.e.*, counteracting gravity) is the **tension** of the rope.

iii) Bodies in motion often feel **frictional forces** (*i.e.*, from surfaces, air, etc.) which retard their motion \implies **frictional force is in the opposite direction of the direction of motion**. If an object does not move on a surface ($\vec{a} = 0$), it may be experiencing a force of **static friction**, \vec{f}_s , with possible values of

$$\boxed{f_s \leq \mu_s n ,} \quad (\text{I-25})$$

$\implies \mu_s \equiv$ coefficient of static friction,
 $n \equiv$ magnitude of the normal force.

Since the object doesn't move under applied force, \vec{F} ,

$$\boxed{\vec{F} - \vec{f}_s = 0 .} \quad (\text{I-26})$$

If one continues to increase the applied force until the object is just on the verge of slipping (*i.e.*, moving), we have reached the maximum of static friction

$$\boxed{f_{s, \max} = \mu_s n .} \quad (\text{I-27})$$

Once the object is in motion, it experiences the force of **kinetic friction**, \vec{f}_k :

$$\boxed{f_k = \mu_k n ,} \quad (\text{I-28})$$

$\implies \mu_k \equiv$ coefficient of kinetic friction.

Note that μ_k does not have to equal μ_s for the same surface. Usually, $\mu_k < \mu_s$ and ranges between 0.01 and 1.5.

iv) Objects that stay suspended in water (neither rise or sink in water), have a **buoyancy force** counteract the weight of the object in the water. This buoyancy force arises from a water pressure differential between the top and bottom of the surface of the object.

b) Thermal Equilibrium.

i) Thermal equilibrium means that an object has the same temperature throughout its interior.

ii) Objects in thermal equilibrium radiate as black-bodies.

iii) For an object to remain in thermal equilibrium is it absorbs energy from the environment, it must emit that same amount of energy that it absorbs.

c) Hydrostatic Equilibrium.

i) Whenever you have a volume of gas, there are two competing forces:

- The force per unit volume from the internal pressure: $F/V = \Delta P \cdot A/V$.
- The weight per unit volume of the gas trying to pull the gas to the lowest potential: $F_g/V = -mg/V = -\rho g$.

ii) Assume we have a column of gas, then $A = \pi r^2$ and $V = \pi r^2 \Delta z$, where r is the radius of the column cylinder and Δz is the length of the column.

Then equating the two forces above, we get

$$\boxed{\frac{\Delta P}{\Delta z} = -\rho g .} \quad (\text{I-29})$$

- This is the equation of **hydrostatic equilibrium** \implies internal pressure is balanced by the weight of the gas.
- We can use a modified version of Form 3 of the ideal gas law $P = \frac{\rho k_B T}{\bar{m}}$, where \bar{m} is the average mass of the gas particles present, to rewrite Eq. (I-29) as

$$\frac{\Delta P}{\Delta z} = -\frac{\bar{m}g}{k_B T} P . \quad (\text{I-30})$$

- This equation can be solved with integral calculus by letting $\Delta z \rightarrow 0$ so that $\Delta P/\Delta z \rightarrow dP/dz$. The solution to this integral equation:

$$\boxed{P(z) = P_\circ e^{-\bar{m}gz/k_B T} ,} \quad (\text{I-31})$$

where P_\circ is the pressure at $z = 0$ and e is the base of the natural logarithm system.

- We could also carried out the integration for number density which results as

$$N = N_\circ e^{-\bar{m}gz/k_B T} , \quad (\text{I-32})$$

this equation is called the **barometric formula** \implies it gives the number of molecules per unit volume as a function of height z .

- d) We could introduce other types of equilibria as well, for example, charge, baryon and lepton conservation in nuclear

reactions, but we will explore those types of equilibria, that is, conservation laws, in a few weeks.

3. Non-equilibria.

- a) We have already seen that for motion, non-equilibrium simply means that something is accelerating following Newton's Second Law of Eq. (I-7).
- b) If hydrostatic equilibrium is not satisfied, then outflows, inflows, and/or winds results.
- c) Here is will concentrate a little on thermal non-equilibrium in gas. When such a condition is met, then one needs to solve the equation of radiative transfer and the net rate equations for all of the quantum transitions that exist for all the species that make up the gas. As you will see, this can get rather complicated.
- d) In a radiation field, the radiant energy flowing per unit time through a surface element $d\sigma$ within a small solid angle $d\Omega$ about a direction defined by the polar angles θ , ϕ (see Figure I-4) within the frequency interval ν to $\nu + d\nu$ is

$$dE = I_\nu(\theta, \phi) d\nu \cos \theta d\sigma d\Omega. \quad (\text{I-33})$$

- e) The **intensity** $I_\nu(\theta, \phi)$ denotes the energy flow per unit time per unit frequency interval per unit solid angle about the direction θ, ϕ across the unit area \perp to this direction [W/m²/Hz/sr in SI units and erg/s/cm²/Hz/sr in cgs units, where sr \equiv steradian].
- i) I_λ as a function of wavelength is related to I_ν via

$$I_\lambda d\lambda = I_\nu d\nu \quad (\text{I-34})$$

or

$$I_\lambda = (c/\lambda^2) I_\nu , \quad (\text{I-35})$$

since $\nu = c/\lambda$.

ii) The total intensity is

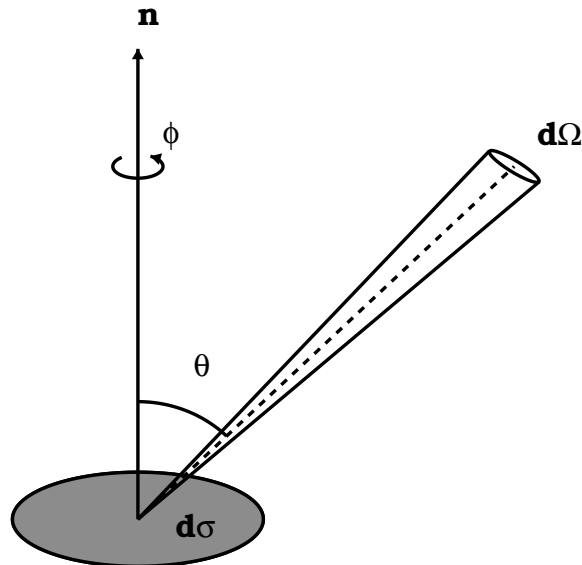
$$I = \int_0^\infty I_\nu d\nu = \int_0^\infty I_\lambda d\lambda. \quad (\text{I-36})$$

iii) It is related to the electric field of the EM wave by the equation

$$I_\nu(z) = \epsilon_0 c |E_\nu(z)|^2 , \quad (\text{I-37})$$

where ϵ_0 is the permittivity of free space, c is the speed of light, and I is the intensity of the electric field at point z in space along the “path” of the EM wave.

iv) Once a photon is emitted into a vacuum (say from a star’s surface), that is, it does not interact with matter during flight, its intensity remains the same at all points along its flight path $\rightarrow I_\nu$ independent of distance r (or d).



- f) The *brightness* or *strength* of light corresponds to the **radiation flux**:

$$\mathcal{F}_\nu = \pi F_\nu = \int_0^{2\pi} \int_{-1}^1 I_\nu(\mu, \phi) \mu d\mu d\phi. \quad (\text{I-38})$$

- i) $\mu = \cos \theta$ and $d\Omega = \sin \theta d\theta d\phi = d\mu d\phi$.

- ii) F_ν is called the **astrophysical flux**.

- iii) In an **isotropic** (*i.e.*, same in all directions), I_ν is independent of θ and $\phi \implies F_\nu = 0$.

- iv) It is often useful to separate F_ν [W/m²/Hz in SI units and erg/s/cm²/Hz in cgs units] into an outward and an inward component in an atmosphere of gas:

$$\pi F_\nu^+ = \int_0^{2\pi} \int_0^1 I_\nu \mu d\mu d\phi \quad (\text{I-39})$$

($0 \leq \mu \leq 1$) outward flux

$$\pi F_\nu^- = \int_0^{2\pi} \int_{-1}^0 I_\nu \mu d\mu d\phi \quad (\text{I-40})$$

($-1 \leq \mu \leq 0$) inward flux,

from which we can write

$$F_\nu = F_\nu^+ + F_\nu^-. \quad (\text{I-41})$$

- v) If I_ν is axisymmetric (*i.e.*, independent of ϕ), then the flux equation becomes:

$$\mathcal{F}_\nu = 2\pi \int_{-1}^1 I_\nu \mu d\mu. \quad (\text{I-42})$$

- vi) The total radiation flux is then given by

$$\mathcal{F} = \int_0^\infty \mathcal{F}_\nu d\nu = \int_0^\infty \mathcal{F}_\lambda d\lambda. \quad (\text{I-43})$$

vii) Note that $4\pi R^2 \mathcal{F}_\nu = L_\nu$ is the monochromatic luminosity, where R is the radius of a spherical object (*e.g.*, a star) emitting the light.

viii) $\mathcal{F}_\nu = \pi I_\nu$ if I_ν is isotropic outward and zero inward.

g) Unlike intensity, flux does scale with distance \implies an object gets fainter the farther away it is:

$$f_\nu = \left(\frac{R_\star}{d}\right)^2 \mathcal{F}_\nu = \frac{1}{4} \alpha_\star^2 \mathcal{F}_\nu . \quad (\text{I-44})$$

i) f_ν is the observed flux of a star of radius R_\star at a distance d .

ii) α_\star is the angular diameter (in radians) of the star as seen at distance d .

h) If a photon is traveling within a medium (*i.e.*, non-vacuum), its intensity does change as it propagates through the gas depending on the opacity of the gas.

i) The **opacity**, χ [cm^{-1}], of a gas measures how opaque the gas is. The opacity of a transition (or continuum) depends primarily on the probability that a transition can occur, which, of course, depends upon the wave functions of the transition.

ii) It is the inverse of the *mean-free-path*, L [cm], of the photon \implies the distance a photon travels before it interacts with another particle.

iii) The opacity dictates how *deep* we can see into a gas. As such, the **optical depth** along depth s (s

increasing outward) of a gas is defined by

$$d\tau_\nu = -\chi_\nu ds , \quad (\text{I-45})$$

τ_ν increases in the opposite direction to s , $\tau_\nu = 0$ at the top and getting larger as you go inward.

iv) One typically does not see deeper into a gas than $\tau_\nu \approx 1$.

v) Typically, opacity is broken up into two components:

$$\chi_\nu = \kappa_\nu + \sigma_\nu , \quad (\text{I-46})$$

where κ_ν is the **absorption opacity** (*i.e.*, once the photon is absorbed, its energy is given up to the thermal pool of the gas via subsequent particle collisions) and σ_ν is the **scattering opacity** (*i.e.*, the photon is re-emitted by the absorbing atom before giving its energy up to the surrounding medium).

i) Consider a rectangular “slab” of gas. As a photon travels through a gas along a small length ds in direction θ ($\mu = \cos \theta$) with respect to a normal line of a rectangular surface, I_ν is attenuated by the following expression:

$$\mu \frac{dI_\nu}{ds} = -\chi_\nu I_\nu . \quad (\text{I-47})$$

j) As previously mentioned, besides this absorption process, particles in the gas can also emit photons (*i.e.*, emission). Hence, I_ν can increase along this path by

$$\mu \frac{dI_\nu}{ds} = \eta_\nu , \quad (\text{I-48})$$

where η_ν [W/m³/Hz/sr in SI units and erg/s/cm³/Hz/sr in cgs units] is the **emissivity** of the gas.

- k) Summing these 2 equations gives the manner in which photons travel through a gas \implies the **Equation of Radiative Transfer**:

$$\mu \frac{dI_\nu}{ds} = \eta_\nu - \chi_\nu I_\nu . \quad (\text{I-49})$$

- l) We can divide each term by χ_ν and get

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu . \quad (\text{I-50})$$

- i) S_ν is called the **source function** and is the ratio of the number of photon creation events divided by the number of photon destruction events ($S_\nu = \eta_\nu/\chi_\nu$) for a given region of gas.

- ii) As such, you can view the transfer equation as the change in intensity of a photon equals the incident intensity on a volume of gas plus any photons created by the gas minus any photons destroyed by the gas.

- iii) Often S_ν will be separated into a *continuum* component and a *line* component: $S_\nu \propto S_\nu^\ell + S_\nu^c$.

- m) For gas in thermodynamic equilibrium, collisional rates exceed radiative (*i.e.*, scattering) rates by large amounts. Hence $\kappa_\nu \gg \sigma_\nu$ in Eq. (I-46) and $\chi_\nu \approx \kappa_\nu$. When this occurs, the emissivity of the gas is given by

$$\eta_\nu = \kappa_\nu B_\nu(T) , \quad (\text{I-51})$$

hence the source function becomes the Planck function for this gas and the transfer equation is somewhat easy to solve:

$$I_\nu = I_{\nu,0} e^{-\tau_\nu/\mu} + \frac{1}{\mu} \int_{\tau_\nu} B_\nu(\tau_\nu) dt . \quad (\text{I-52})$$

n) For non-thermodynamic equilibrium gas, however, S_ν is much more difficult to ascertain, since it will depend upon both the mean intensity ($J_\nu = \frac{1}{2} \int I_\nu d\mu$) of the photons and the thermal nature of the gas in the volume of interest \implies the equation of transfer becomes a *integral-differential equation* in I_ν !

i) The differential portion of the radiative transfer is obvious from Eq. (I-50). The integral portion comes from the expression of the source function in these cases:

$$S_\nu = \frac{r_\nu}{\phi_\nu + r_\nu} S_\nu^c + \frac{\phi_\nu}{\phi_\nu + r_\nu} S_\nu^\ell, \quad (\text{I-53})$$

where S_ν^c is the continuum source function, S_ν^ℓ is the line source function,

$$r_\nu = \frac{\chi_\nu^c}{\chi_{\nu_0}^\ell} \quad (\text{I-54})$$

is the fraction of the continuous opacity, χ_ν^c , over the line center opacity, $\chi_{\nu_0}^\ell$,

$$\phi_\nu = \frac{\chi_\nu^\ell}{\chi_{\nu_0}^\ell} \quad (\text{I-55})$$

is the profile function (usually a Gaussian, Lorentzian, or Voigt [combination of the first two] profile) of the line which is the fraction of the line opacity at some portion of the profile, χ_ν^ℓ , over the line center opacity, $\chi_{\nu_0}^\ell$.

ii) The *continuum* source function is determined by

$$S_\nu^c = \frac{\kappa_\nu B_\nu + \sigma_\nu J_\nu}{\kappa_\nu + \sigma_\nu} = \epsilon_\nu^c B_\nu + (1 - \epsilon_\nu^c) J_\nu, \quad (\text{I-56})$$

where J_ν is the angle-averaged mean intensity (*i.e.*, an integral over angle θ — see previous page), and

$\epsilon_\nu^c = \kappa_\nu / (\kappa_\nu + \sigma_\nu)$ is the thermal fraction of the continuum emission.

- iii) The *line* source function is somewhat more difficult to determine in non-equilibrium cases:

$$\begin{aligned} S_\nu^\ell &= (1 - \epsilon_\nu^\ell) \int \phi_\nu J_\nu d\nu + \epsilon_\nu^\ell B_\nu \\ &= (1 - \epsilon_\nu^\ell) \bar{J} + \epsilon_\nu^\ell B_\nu, \end{aligned} \quad (\text{I-57})$$

where \bar{J} is the profile-integrated mean intensity, and ϵ_ν^ℓ is the thermal fraction of the line emission.

- iv) The line source function also can be determined from

$$S_\nu^\ell = \frac{2h\nu^3}{c^2} \left(\frac{n_i g_j}{n_j g_i} - 1 \right)^{-1}, \quad (\text{I-58})$$

where n_i and n_j are the particle number densities of the lower ('*i*') and upper ('*j*') energy states, respectively, and g_i and g_j are their corresponding *statistical weights* which are calculated from quantum numbers. In this equation, the number density ratio is determined from the radiative and collisional rate equations and the solution to radiative rates equation require integrals to be solved (see below). Note that Eq. (I-58) reduces to the Planck function is the particle density ratio achieves its equilibrium value.

- o) Now we have to worry about the net rate equations. Let N_i be the number of atoms in a sample of gas who have electrons in a lower-energy state and let N_j be the number in the upper-energy state. If we ignore matter particle collisions, the rate in which the upper level either gains

or loses an electron to the lower level is given by

$$\frac{dN_j}{dt} = -N_j A_{ji} - N_j B_{ji} \rho(\nu_{ij}) + N_i B_{ij} \rho(\nu_{ij}) . \quad (\text{I-59})$$

- i) A_{ji} represents the spontaneous emission rate. The number of particles leaving this state, per unit time, by this mechanism is $N_j A_{ji}$.

- ii) The transition rate for stimulated emission is proportional to the energy density of the EM field, ρ , whose energy corresponds to the energy difference of the two states, $h\nu_{ij}$. The number of particles leaving this state, per unit time, by this mechanism is $N_j B_{ji} \rho(\nu_{ij})$.

- iii) The transition rate for absorption from the lower level to the upper level also is proportional to the energy density of the electromagnetic field of energy $h\nu_{ij}$. The number of particles entering the upper state from the lower state, per unit time, is $N_i B_{ij} \rho(\nu_{ij})$.

- iv) Remember that ρ is the radiation field energy density which depends upon the integral of the radiation intensity. As such, determining the values of the Einstein- B terms requires *cross-talk* with the radiative transfer equation.

- p) This is as far as we need to go here. I just wanted to demonstrate that carrying out non-equilibrium physics is very complicated. This is the main reason why scientists usually work on the equilibrium cases first, they are much simpler to solve.