PHYS-4617: Quantum Physics
Problem Set 1 (Due: 10 February 2005)

1. (40 pts) For the following questions, the Lyman-α line at 1215.671 Å (vacuum wavelength) is a transition from the ground $1s$ state, with a total angular momentum quantum number vector of $J = 1/2$, to the first excited $2p$ state with $J = 3/2$. It has an oscillator strength (times the statistical weight) of $g_i f_{ij} = 0.5549$ (in cgs units). Meanwhile, the Lyman-β line has $g_i f_{ij} = 0.1055$ (cgs units) for the $1s - 3p$ ($J = 1/2$ to $J = 3/2$) transition at 1025.723 Å (vacuum). Finally, the Hα line at 6564.626 Å (vacuum wavelength) has $g_i f_{ij} = 0.5798$ (cgs units) for the $2s$ $J = 1/2$ to the $3p$ $J = 3/2$ transition.

   (a) What are the cgs units of f_{ij}? (*Hint*: Use dimensional analysis in Eq. (I-65) and note that statistical weights are unitless. Also note that you will have to figure out how to express 'esu' in terms of grams, centimeters, and seconds.)

   (b) What are the Einstein A-values for these three transitions? (*Hint*: See Eq. (I-65) and Table (I-2) in the notes to help you calculate this.)

   (c) What is the approximate level thickness (in eVs) for these three states?

   (d) Calculate the population ratio between the $2p$ and $1s$ states, and the $3p$ and $2s$ states for both a 10,000 K gas and a 3000 K gas assuming the hydrogen is in thermal equilibrium.

2. (30 pts) The three lowest energy levels of Mg II are associated with the following two electron configurations: $1s^2 2s^2 2p^6 3s^2$ (the ground state which we will call state '1' here) and $1s^2 2s^2 2p^6 3s3p$ (which has both a triplet state, call this state '2' here, and a singlet state, call this state '3' here).

   (a) What is the spectroscopic notation of each of these states?

   (b) Which transitions of these three states would be considered *allowed*, *semi-forbidden*, and/or *forbidden*? List all reasons why this is the case for each transition.

3. (20 pts) Consider the **Gaussian** distribution

$$\rho(x) = A\, e^{-\lambda\,(x-a)^2},$$

   where A, a, and λ are constants. (Look up any integrals you need.)

   (a) Use the normalization integral of the probability density,

$$\int_{-\infty}^{+\infty} \rho(x)\, dx = 1,$$

   to determine A.

(b) Find $\langle x \rangle$, $\langle x^2 \rangle$, and σ.

(c) Sketch the graph of $\rho(x)$.

4. (40 pts) Consider the wave function

$$\Psi(x,t) = A\,e^{-\lambda|x|}\,e^{-i\omega t}\ ,$$

where A, ω, and λ are positive real constants. (Look up any integrals you need.)

(a) Normalize Ψ.

(b) Determine the expectation values of x and x^2.

(c) Find the standard deviation of x. Sketch the graph of $|\Psi|^2$ as a function of x, and mark the points $(\langle x \rangle + \sigma)$ and $(\langle x \rangle - \sigma)$ to illustrate the sense in which σ represents the "spread" in x. What is the probability that the particle would be found outside this range?