

**PHYS-4617: Quantum Physics**  
**Problem Set 3 (Due: 6 November 2003)**

1. (10 pts) Prove the triangle inequality:  $\|(|\alpha\rangle + |\beta\rangle)\| \leq \|\alpha\| + \|\beta\|$ .

2. (30 pts) Let

$$\mathbf{T} = \begin{pmatrix} 1 & 1 - i \\ 1 + i & 0 \end{pmatrix}.$$

(a) Verify that  $\mathbf{T}$  is Hermitian.

(b) Find its eigenvalues (note that they are real).

(c) Find and normalize the eigenvectors (note that they are orthogonal).

3. (30 pts) For the following questions, start with the equation:

$$\frac{d}{dt}\langle Q \rangle = \frac{i}{\hbar}\langle [\hat{H}, \hat{Q}] \rangle + \left\langle \frac{\partial \hat{Q}}{\partial t} \right\rangle.$$

(a) Show that

$$\frac{d}{dt}\langle xp \rangle = 2\langle T \rangle - \left\langle x \frac{dV}{dx} \right\rangle,$$

where  $T$  is the kinetic energy ( $H = T + V$ ) results from this equation.

(b) In a stationary state, show that the left hand side of the equation in (a) is zero which results in the **virial theorem**:

$$2\langle T \rangle = \left\langle x \frac{dV}{dx} \right\rangle.$$

(c) Use the virial theorem to prove that  $\langle T \rangle = \langle V \rangle$  for the stationary states of a harmonic oscillator.