PHYS-4617: Quantum Physics
Problem Set 4 (Due: 25 November 2003)

1. (30 pts) For the hydrogen atom

   (a) Normalize R_{20} and construct the function ψ_{200}.

   (b) Normalize R_{21} and construct ψ_{210}, ψ_{211}, and ψ_{21-1}.

2. (50 pts) The following questions concern the orbital angular momentum operator.

   (a) Starting with the canonical commutation relations for position and momentum,

   $$[r_i, p_j] = -[p_i, r_j] = i\hbar\delta_{ij} \ , \quad [r_i, r_j] = [p_i, p_j] = 0 \ ,$$

   work out the following commutators:

   $$[L_z, x] = i\hbar y, \quad [L_z, y] = -i\hbar x, \quad [L_z, z] = 0,$$
   $$[L_z, p_x] = i\hbar p_y, \quad [L_z, p_y] = -i\hbar p_x, \quad [L_z, p_z] = 0.$$

   (b) Use these results to obtain $[L_z, L_x] = i\hbar L_y$ directly from Eq. (VI-2).

   (c) Evaluate the commutators $\left[L_z, r^2\right]$ and $\left[L_z, p^2\right]$, where $r^2 = x^2 + y^2 + z^2$ and $p^2 = p_x^2 + p_y^2 + p_z^2$.

   (d) Show that the Hamiltonian $H = (p^2/2m) + V$ commutes with all three components of $\mathbf{L}$, provided that V depends only upon r. (Thus H, L^2, and L_z are mutually compatible observables.)